- MAT112 Prof. Kelly / T.A. Pabon *Recitation will start soon.* We will pass this course with a great grade! We will meet our academic and professional goals!

- MAT112 T.A. Pabon

We will be courteous, civil to each other.

NO SUCH THING AS OBVIOUS QUESTION

ask ask any doubts to clear up

- MAT112 T.A. Pabon

-Attendance policy.
-Recitation Worksheet
Problems to be graded will be
posted on Canvas.

Review (Test)

① Geometric Series Test $\sum ar^n$

$|r|<1$, then converge to $\dfrac{a_0}{1-r}$

$|r| \geq 1$, div

② P-Test $\sum \dfrac{1}{n^p}$ $p>1$ con, $p \leq 1$ div

③ Nth Term D.T. $\sum a_n$

$\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ div.

④ Integral Test (i) Let $a_n = f(x)$

continuous, positive, decreasing

(ii) $\int_N^\infty f(n)\,dx$ con $\rightarrow \sum a_n$ con

div $\rightarrow \sum a_n$ div

⑤ D.C.T (B.C → S.C)

(S.D → B.D)

⑥ L.C.T (i) $\lim_{n\to\infty} \dfrac{a_n}{b_n} = L$, both C or both D

(ii) $= 0$ & $\sum b_n$ con $\rightarrow \sum a_n$ con

(iii) $= \infty$ & $\sum b_n$ div $\rightarrow \sum a_n$ div

Review (Test)

① Geometric Series Test $\sum ar^n$

$|r|<1$, then converge to $\frac{a_0}{1-r}$

$|r| \geq 1$, div

② P-Test $\sum \frac{1}{n^p}$ $p>1$ con, $p \leq 1$ div

③ Nth Term DT $\sum a_n$

$\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ div

④ Integral Test (i) Let $a_n = f(x)$ continuous, positive, decreasing

(ii) $\int_N^{\infty} f(x)dx$ con $\to \sum a_n$ con, div $\to \sum a_n$ div

⑤ DCT (BC → SC)

(SD → BD)

⑥ LCT (i) $\lim_{n\to\infty} \frac{a_n}{b_n} = L$, both C or both D

(ii) $= 0$ & $\sum b_n$ con $\to \sum a_n$ con

(iii) $= \infty$ & $\sum b_n$ div $\to \sum a_n$ div

10.5 Absolute Convergence Test ; Ratio & Root Test

Def If $\sum |a_n|$ converges, then $\sum a_n$ converges absolutely

⑦ Thm 12 (Absolute Convergence Test)

$\boxed{\text{If } \sum |a_n| \text{ converges, then } \sum a_n \text{ converges}}$

Ex 1] $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \cdots$

$\sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{1}{n^2} \right| = \sum_{n=1}^{\infty} \frac{1}{n^2}$ con by P-Test with $P=2>1$

Thus, I con by ACT

10.5 Absolute Convergence Test; Ratio & Root Test

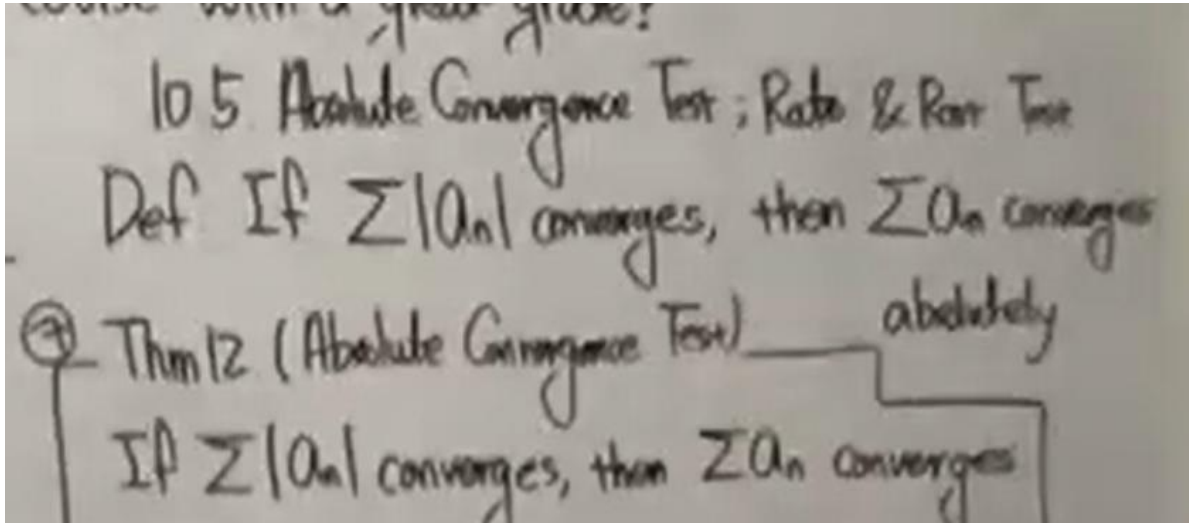
Def If $\sum |a_n|$ converges, then $\sum a_n$ converges absolutely

Thm 12 (Absolute Convergence Test)

If $\sum |a_n|$ converges, then $\sum a_n$ converges

1. Determine if the following series is absolutely convergent, conditionally convergent or divergent.

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n^3+1}$$



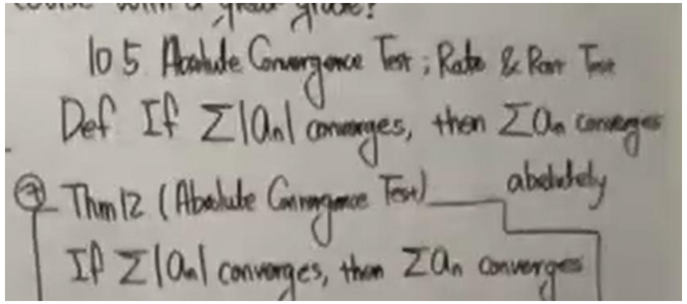
Okay, let's first see if the series converges or diverges if we put absolute value on the series terms.

$$\sum_{n=2}^{\infty} \left| \frac{(-1)^{n+1}}{n^3+1} \right| = \sum_{n=2}^{\infty} \frac{1}{n^3+1}$$

Now, notice that,

$$\frac{1}{n^3+1} < \frac{1}{n^3}$$

Okay, let's first see if the series converges or diverges if we put absolute value on the series terms.



$$\sum_{n=2}^{\infty}\left|\frac{(-1)^{n+1}}{n^3+1}\right| = \sum_{n=2}^{\infty}\frac{1}{n^3+1}$$

Now, notice that,

$$\frac{1}{n^3+1} < \frac{1}{n^3}$$

and we know by the p-series test that

$$\sum_{n=2}^{\infty}\frac{1}{n^3}$$

converges.

Therefore, by the Comparison Test we know that the series from the problem statement,

$$\sum_{n=2}^{\infty}\frac{1}{n^3+1}$$

will also converge.

Hide Step 2 ▾

So, because the series with the absolute value converges we know that the series in the problem statement is **absolutely convergent**.

3. Determine if the following series is absolutely convergent, conditionally convergent or divergent.

$$\sum_{n=3}^{\infty} \frac{(-1)^{n+1}(n+1)}{n^3+1}$$

10.5 Absolute Convergence Test ; Ratio & Root Test

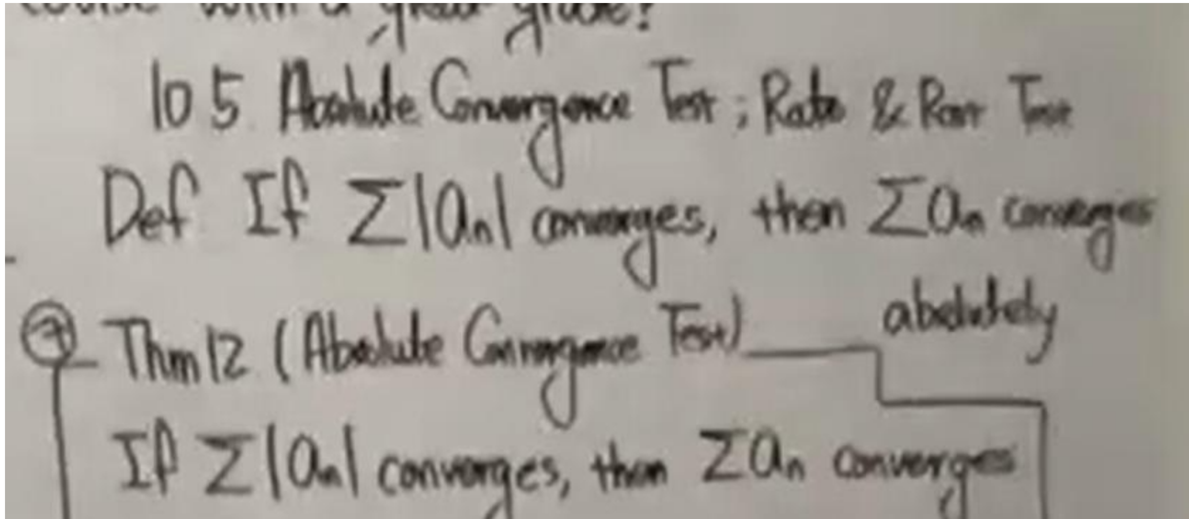
Def If $\sum |a_n|$ converges, then $\sum a_n$ converges absolutely

Thm 12 (Absolute Convergence Test)

If $\sum |a_n|$ converges, then $\sum a_n$ converges

Okay, let's first see if the series converges or diverges if we put absolute value on the series terms.

$$\sum_{n=3}^{\infty} \left| \frac{(-1)^{n+1}(n+1)}{n^3+1} \right| = \sum_{n=3}^{\infty} \frac{n+1}{n^3+1}$$



Okay, let's first see if the series converges or diverges if we put absolute value on the series terms.

$$\sum_{n=3}^{\infty} \left| \frac{(-1)^{n+1}(n+1)}{n^3+1} \right| = \sum_{n=3}^{\infty} \frac{n+1}{n^3+1}$$

We know by the p-series test that the following series converges.

$$\sum_{n=3}^{\infty} \frac{1}{n^2}$$

10.5 Absolute Convergence Test ; Ratio & Root Test

Def If $\sum |a_n|$ converges, then $\sum a_n$ converges absolutely

Thm 12 (Absolute Convergence Test)

If $\sum |a_n|$ converges, then $\sum a_n$ converges

Okay, let's first see if the series converges or diverges if we put absolute value on the series terms.

$$\sum_{n=3}^{\infty}\left|\frac{(-1)^{n+1}(n+1)}{n^3+1}\right| = \sum_{n=3}^{\infty}\frac{n+1}{n^3+1}$$

We know by the p-series test that the following series converges.

$$\sum_{n=3}^{\infty}\frac{1}{n^2}$$

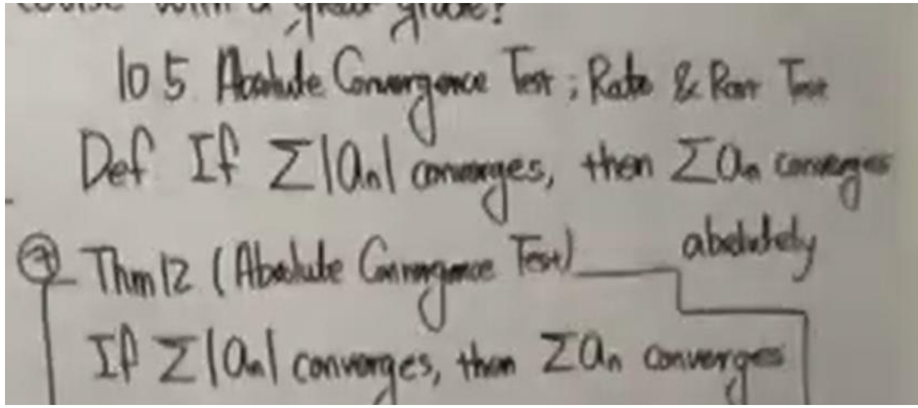
If we now compute the following limit,

$$c = \lim_{n\to\infty}\left[\frac{n+1}{n^3+1}\,\frac{n^2}{1}\right] = \lim_{n\to\infty}\left[\frac{n^3+n^2}{n^3+1}\right] = 1$$

we know by the Limit Comparison Test that the two series in this problem have the same convergence because c is neither zero or infinity and because $\sum\limits_{n=3}^{\infty}\frac{1}{n^2}$ converges we know that the series from the problem statement must also converge.

Hide Step 2 ▾

So, because the series with the absolute value converges we know that the series in the problem statement is **absolutely convergent**.



10.5 Absolute Convergence Test; Ratio & Root Tests

Def If $\sum |a_n|$ converges, then $\sum a_n$ converges

Thm 12 (Absolute Convergence Test) absolutely

$\boxed{\text{If } \sum |a_n| \text{ converges, then } \sum a_n \text{ converges}}$

3/26/R I will pass this course with a great grade!

[Ex2] $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2} = I$

$\sum_{n=1}^{\infty} \left| \frac{\sin(n)}{n^2} \right|$ con. by D.C.T.

$\frac{0}{n^2} \leq \frac{|\sin(n)|}{n^2} \leq \frac{1}{n^2}$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ con by P-Test with $P=2>1$

Thus, I con by A.C.T

Thm 13. (Ratio Test)

Let $\sum a_n$ be any series and $r = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

Then ① If $r<1$, then $\sum a_n$ converges (absolutely)

② If $r>1$, then $\sum a_n$ diverges.

③ If $r=1$, then the test fails.

[Ex3] $\sum_{n=0}^{\infty} \frac{2^n+5}{3^n}$ $\quad \frac{2^{n+1}}{2^n} = \frac{\cancel{2^n}\,2}{\cancel{2^n}} = 2 \quad \frac{3^n}{3^{n+1}} = \frac{\cancel{3^n}}{\cancel{3^n}\cdot 3}$

$r = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(2^{n+1}+5)}{\underbrace{3^{n+1}}_{3}} \cdot \frac{3^n}{(2^n+5)} \right|$

$= \frac{1}{3} \lim_{n\to\infty} \frac{2^{n+1}+5}{2^n+5} = \frac{1}{3} \lim_{n\to\infty} \frac{2+\frac{5}{2^n}}{1+\frac{5}{2^n}} = \frac{2}{3} < 1$

Thus, I con by Ratio Test

(absolutely)

3/26/R I will pass this course with a great grade!

[Ex4] $\sum_{n=1}^{\infty} \frac{(2n)!}{n!\,n!}$ $\quad \frac{5!}{3!} = \frac{5\cdot 4\cdot \cancel{3!}}{\cancel{3!}}$

$r = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

$= \lim_{n\to\infty} \left| \frac{(2(n+1))!}{(n+1)!\,(n+1)!} \cdot \frac{n!\,n!}{(2n)!} \right|$ $\quad (2n+2)!$

$= \lim_{n\to\infty} \left| \frac{(2n+2)(2n+1)\cancel{(2n)!}}{(n+1)\cancel{n!}(n+1)\cancel{n!}} \cdot \frac{\cancel{n!}\,\cancel{n!}}{\cancel{(2n)!}} \right|$

$= \lim_{n\to\infty} \frac{(2n+2)(2n+1)}{(n+1)(n+1)} = \frac{4}{1} = 4 > 1$

Thus, I div. by Ratio Test.

Thm 13. (Ratio Test)

Let $\sum a_n$ be any series and $r = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

Then. ① If $r<1$, then $\sum a_n$ converges (absolutely)

② If $\boxed{r>1}$ then $\sum a_n$ diverges.

③ If $r=1$, then the test fails.

$\frac{3^n}{3^{n+1}} = \frac{\cancel{3^n}}{\cancel{3^n}\,3}$

[Ex3]. $\sum_{n=0}^{\infty} \frac{2^n+5}{3^n}$ $\quad \frac{2^{n+1}}{2^n} = \frac{\cancel{2^n}\,2}{\cancel{2^n}} = 2$

$r = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(2^{n+1}+5)}{\cancel{3^{n+1}}\,3} \cdot \frac{\cancel{3^n}}{(2^n+5)} \right|$

$= \frac{1}{3} \lim_{n\to\infty} \frac{2^{n+1}+5}{2^n+5} = \frac{1}{3} \lim_{n\to\infty} \frac{2+\frac{5}{2^n}}{1+\frac{5}{2^n}} = \frac{2}{3} < 1$ $\quad (\frac{5}{2^n} \to 0)$

Thus, I con. by Ratio Test

(absolutely)

3. Determine if the following series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{(-2)^{1+3n}(n+1)}{n^2 5^{1+n}}$$

course with a great grade?

Thm 13. (Ratio Test)

Let $\sum a_n$ be any series and $r = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

Then. ① If $r<1$, then $\sum a_n$ converges (absolutely)

② If $\boxed{r>1}$ then $\sum a_n$ diverges.

③ If $r=1$, then the test fails.

3^n

course with a great grade?

Thm 13. (Ratio Test)

Let $\sum a_n$ be any series and $r = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

Then. ① If $r<1$, then $\sum a_n$ converges (absolutely)

② If $\boxed{r>1}$ then $\sum a_n$ diverges.

③ If $r=1$, then the test fails.

3. Determine if the following series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{(-2)^{1+3n}(n+1)}{n^2 5^{1+n}}$$

We'll need to compute L.

$$L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|a_{n+1}\frac{1}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-2)^{1+3(n+1)}(n+1+1)}{(n+1)^2 5^{1+n+1}}\frac{n^2 5^{1+n}}{(-2)^{1+3n}(n+1)}\right|$$

3. Determine if the following series converges or diverges.

We'll need to compute L.

$$L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|a_{n+1}\frac{1}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-2)^{1+3(n+1)}(n+1+1)}{(n+1)^2 5^{1+n+1}}\,\frac{n^2 5^{1+n}}{(-2)^{1+3n}(n+1)}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(-2)^{4+3n}(n+2)}{(n+1)^2 5^{2+n}}\,\frac{n^2 5^{1+n}}{(-2)^{1+3n}(n+1)}\right| = \lim_{n\to\infty}\left|\frac{(-2)^3(n+2)}{(n+1)^2(5)}\,\frac{n^2}{(n+1)}\right|$$

$$= \lim_{n\to\infty}\left|\frac{-8n^2(n+2)}{5(n+1)^2(n+1)}\right| = \frac{8}{5}$$

When computing a_{n+1} be careful to pay attention to any coefficients of n and powers of n. Failure to properly deal with these is one of the biggest mistakes that students make in this computation and mistakes at that level often lead to the wrong answer!

Hide Step 2 ▾

Okay, we can see that $L = \frac{8}{5} > 1$ and so by the Ratio Test the series **diverges**.

course with a great grade?

Thm 13. (Ratio Test)

Let $\sum a_n$ be any series and $r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$

Then ① If $r<1$, then $\sum a_n$ converges (absolutely)

② If $\boxed{r>1}$ then $\sum a_n$ diverges.

③ If $r=1$, then the test fails.

course with a great grade?

Thm 13. (Ratio Test)

Let $\sum a_n$ be any series and $r = \lim\limits_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

Then. ① If $r<1$, then $\sum a_n$ converges (absolutely)

② If $\boxed{r>1}$ then $\sum a_n$ diverges.

③ If $r=1$, then the test fails.

$$\frac{5^{n+1}}{5^{n+2}} = 5^{(n+1)-(n+2)} = 5^{-1} = \frac{1}{5}\ .$$

$$\frac{-2^{4+3n}}{-2^{1+3n}} = -2^{(4+3n)-(1+3n)} = -2^3.$$

3. Determine if the following series converges or diverges.

We'll need to compute L.

$$L=\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\left|a_{n+1}\frac{1}{a_n}\right|=\lim_{n\to\infty}\left|\frac{(-2)^{1+3(n+1)}(n+1+1)}{(n+1)^2 5^{1+n+1}}\,\frac{n^2 5^{1+n}}{(-2)^{1+3n}(n+1)}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(-2)^{4+3n}(n+2)}{(n+1)^2 5^{2+n}}\,\frac{n^2 5^{1+n}}{(-2)^{1+3n}(n+1)}\right|=\lim_{n\to\infty}\left|\frac{(-2)^3(n+2)}{(n+1)^2(5)}\,\frac{n^2}{(n+1)}\right|$$

$$=\lim_{n\to\infty}\left|\frac{-8n^2(n+2)}{5(n+1)^2(n+1)}\right|=\frac{8}{5}$$

When computing a_{n+1} be careful to pay attention to any coefficients of n and powers of n. Failure to properly deal with these is one of the biggest mistakes that students make in this computation and mistakes at that level often lead to the wrong answer!

Hide Step 2 ▾

Okay, we can see that $L=\frac{8}{5}>1$ and so by the Ratio Test the series **diverges**.

course with a great grade?

Thm13. (Ratio Test)

Let $\sum a_n$ be any series and $r=\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$

Then. ① If $r<1$, then $\sum a_n$ converges (absolutely)

② If $\boxed{r>1}$ then $\sum a_n$ diverges.

③ If $r=1$, then the test fails. 3^n

Convergence or Divergence

Which of the series in Exercises 57–64 converge, and which diverge? Give reasons for your answers.

57. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n! n!}{(2n)!}$

58. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (3n)!}{n!(n+1)!(n+2)!}$

course with a great grade?

Thm 13. (Ratio Test)

Let $\sum a_n$ be any series and $r = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

Then. ① If $r<1$, then $\sum a_n$ converges (absolutely)

② If $\boxed{r>1}$ then $\sum a_n$ diverges.

③ If $r=1$, then the test fails.

3^n

Convergence or Divergence

Which of the series in Exercises 57–64 converge, and which diverge? Give reasons for your answers.

57. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n! n!}{(2n)!}$

58. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (3n)!}{n!(n+1)!(n+2)!}$

57. converges by the Ratio Test: $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{2^{n+1}(n+1)!(n+1)!}{(2n+2)!}\cdot\frac{(2n)!}{2^n n! n!}$:

course with a great grade?

Thm 13. (Ratio Test)

Let $\sum a_n$ be any series and $r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$

Then. ① If $r<1$, then $\sum a_n$ converges (absolutely)

② If $\boxed{r>1}$ then $\sum a_n$ diverges.

③ If $r=1$, then the test fails.

3^n

Convergence or Divergence

Which of the series in Exercises 57–64 converge, and which diverge? Give reasons for your answers.

57. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n! n!}{(2n)!}$

58. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (3n)!}{n!(n+1)!(n+2)!}$

57. converges by the Ratio Test: $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{2^{n+1}(n+1)!(n+1)!}{(2n+2)!}\cdot\frac{(2n)!}{2^n n! n!} = \lim_{n\to\infty}\frac{2(n+1)(n+1)}{(2n+2)(2n+1)}$

course with a great grade?

Thm 13. (Ratio Test)

Let $\sum a_n$ be any series and $r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$

Then. ① If $r<1$, then $\sum a_n$ converges (absolutely)

② If $\boxed{r>1}$ then $\sum a_n$ diverges.

③ If $r=1$, then the test fails.

3^n

Convergence or Divergence

Which of the series in Exercises 57–64 converge, and which diverge? Give reasons for your answers.

57. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n! n!}{(2n)!}$

58. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (3n)!}{n!(n+1)!(n+2)!}$

57. converges by the Ratio Test: $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{2^{n+1}(n+1)!(n+1)!}{(2n+2)!}\cdot\frac{(2n)!}{2^n n! n!} = \lim_{n\to\infty} \frac{2(n+1)(n+1)}{(2n+2)(2n+1)} = \lim_{n\to\infty} \frac{n+1}{2n+1} = \frac{1}{2} < 1$

course with a great grade?

Thm 13. (Ratio Test)

Let $\sum a_n$ be any series and $r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$

Then. ① If $r<1$, then $\sum a_n$ converges (absolutely)

② If $\boxed{r>1}$ then $\sum a_n$ diverges.

③ If $r=1$, then the test fails.

3^n

Convergence or Divergence

Which of the series in Exercises 57–64 converge, and which diverge? Give reasons for your answers.

57. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n! n!}{(2n)!}$ **58.** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (3n)!}{n!(n+1)!(n+2)!}$

58. diverges by the Ratio Test: $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(3n+3)!}{(n+1)!(n+2)!(n+3)!}\cdot\frac{n!(n+1)!(n+2)!}{(3n)!}$

course with a great grade?

Thm 13. (Ratio Test)

Let $\sum a_n$ be any series and $r = \displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$

Then. ① If $r<1$, then $\sum a_n$ converges (absolutely)

② If $\boxed{r>1}$ then $\sum a_n$ diverges.

③ If $r=1$, then the test fails.

3^n

Convergence or Divergence

Which of the series in Exercises 57–64 converge, and which diverge? Give reasons for your answers.

57. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n! n!}{(2n)!}$

58. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (3n)!}{n!(n+1)!(n+2)!}$

58. diverges by the Ratio Test: $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{(3n+3)!}{(n+1)!(n+2)!(n+3)!}\cdot\frac{n!(n+1)!(n+2)!}{(3n)!} = \lim_{n\to\infty}\frac{(3n+3)(3+2)(3n+1)}{(n+1)(n+2)(n+3)}$

course with a great grade?

Thm 13. (Ratio Test)

Let $\sum a_n$ be any series and $r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$

Then. ① If $r<1$, then $\sum a_n$ converges (absolutely)

② If $\boxed{r>1}$ then $\sum a_n$ diverges.

③ If $r=1$, then the test fails.

3^n

Convergence or Divergence

Which of the series in Exercises 57–64 converge, and which diverge? Give reasons for your answers.

57. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n! n!}{(2n)!}$

58. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (3n)!}{n!(n+1)!(n+2)!}$

58. diverges by the Ratio Test: $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{(3n+3)!}{(n+1)!(n+2)!(n+3)!}\cdot\frac{n!(n+1)!(n+2)!}{(3n)!} = \lim_{n\to\infty}\frac{(3n+3)(3+2)(3n+1)}{(n+1)(n+2)(n+3)}$

$\displaystyle = \lim_{n\to\infty} 3\left(\frac{3n+2}{n+2}\right)\left(\frac{3n+1}{n+3}\right) = 3\cdot 3\cdot 3 = 27 > 1$

course with a great grade?

Thm 13. (Ratio Test)

Let $\sum a_n$ be any series and $r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$

Then. ① If $r<1$, then $\sum a_n$ converges (absolutely)

② If $\boxed{r>1}$ then $\sum a_n$ diverges.

③ If $r=1$, then the test fails.

3^n

3/26/R I will pass this course with a great grade!

[Ex6] $\sum_{n=1}^{\infty} \left(\frac{1}{1+n}\right)^n$

$$\rho = \lim_{n\to\infty} \sqrt[n]{\left|\frac{1}{1+n}\right|^n}$$

$$= \lim_{n\to\infty} \frac{1}{1+n} = 0 < 1$$

Thus, I con. (absolutely) by root test

⑨ Thm 14 (Root Test)

Let $\sum a_n$ be any series and $\rho = \lim_{n\to\infty} \sqrt[n]{|a_n|}$

Then ① If $\rho < 1$ then $\sum a_n$ converges (absolutely)

② If $\rho > 1$ then $\sum a_n$ diverges

③ If $\rho = 1$ then the test fails

[Ex5] $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

$$\rho = \lim_{n\to\infty} \sqrt[n]{\left|\frac{n^2}{2^n}\right|} = \lim_{n\to\infty} \sqrt[n]{\frac{n^2}{2^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^2}}{\sqrt[n]{2^n}}$$

Thm 5 (No. 1)

$$= \lim_{n\to\infty} \frac{(\sqrt[n]{n})^2}{2} = \frac{1^2}{2} = \frac{1}{2} < 1$$

Thus, I con. by root test

Using the Root Test

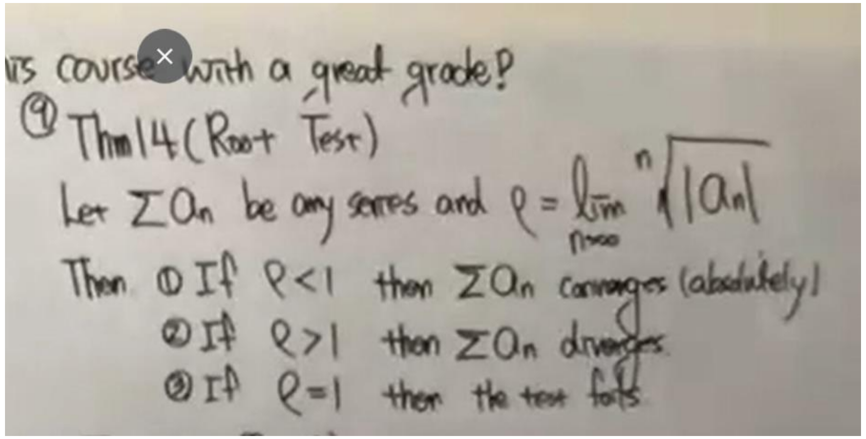
In Exercises 9–16, use the Root Test to determine if each series converges absolutely or diverges.

9. $\displaystyle\sum_{n=1}^{\infty} \frac{7}{(2n+5)^n}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{(3n)^n}$

11. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-5}\right)^n$

12. $\displaystyle\sum_{n=1}^{\infty} \left(-\ln\left(e^2+\frac{1}{n}\right)\right)^{n+1}$



Using the Root Test

In Exercises 9–16, use the Root Test to determine if each series converges absolutely or diverges.

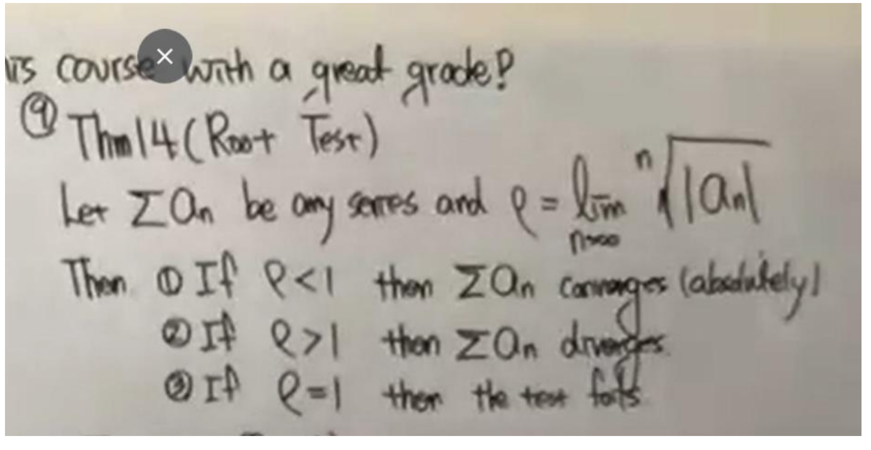
9. $\displaystyle\sum_{n=1}^{\infty} \frac{7}{(2n+5)^n}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{(3n)^n}$

11. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-5}\right)^n$

12. $\displaystyle\sum_{n=1}^{\infty} \left(-\ln\left(e^2+\frac{1}{n}\right)\right)^{n+1}$

11. $$\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{4n+3}{3n-5}\right)^n\right|} =$$



Using the Root Test

In Exercises 9–16, use the Root Test to determine if each series converges absolutely or diverges.

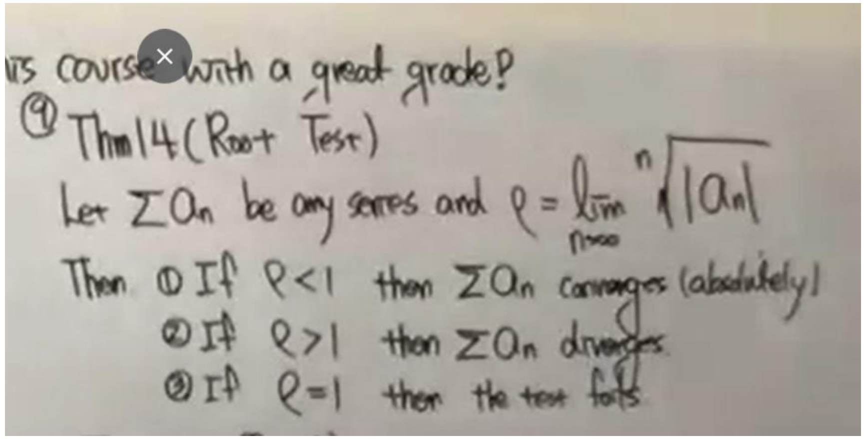
9. $\displaystyle\sum_{n=1}^{\infty} \frac{7}{(2n+5)^n}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{(3n)^n}$

11. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-5}\right)^n$

12. $\displaystyle\sum_{n=1}^{\infty} \left(-\ln\left(e^2+\frac{1}{n}\right)\right)^{n+1}$

$$11.\quad \lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{4n+3}{3n-5}\right)^n\right|} = \lim_{n\to\infty}\left(\frac{4n+3}{3n-5}\right)$$



Using the Root Test

In Exercises 9–16, use the Root Test to determine if each series converges absolutely or diverges.

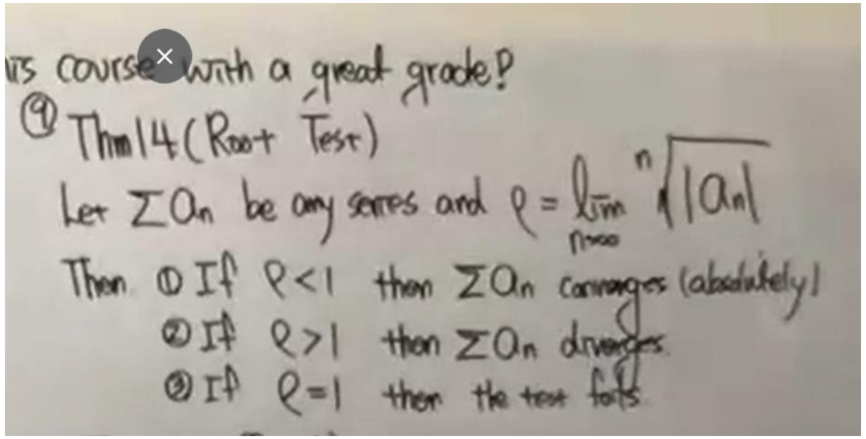
9. $\displaystyle\sum_{n=1}^{\infty} \frac{7}{(2n+5)^n}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{(3n)^n}$

11. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-5}\right)^n$

12. $\displaystyle\sum_{n=1}^{\infty} \left(-\ln\left(e^2+\frac{1}{n}\right)\right)^{n+1}$

$$11. \quad \lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{4n+3}{3n-5}\right)^n\right|} = \lim_{n\to\infty} \left(\frac{4n+3}{3n-5}\right) = \lim_{n\to\infty} \left(\frac{4}{3}\right) = \frac{4}{3} > 1$$



Using the Root Test

In Exercises 9–16, use the Root Test to determine if each series converges absolutely or diverges.

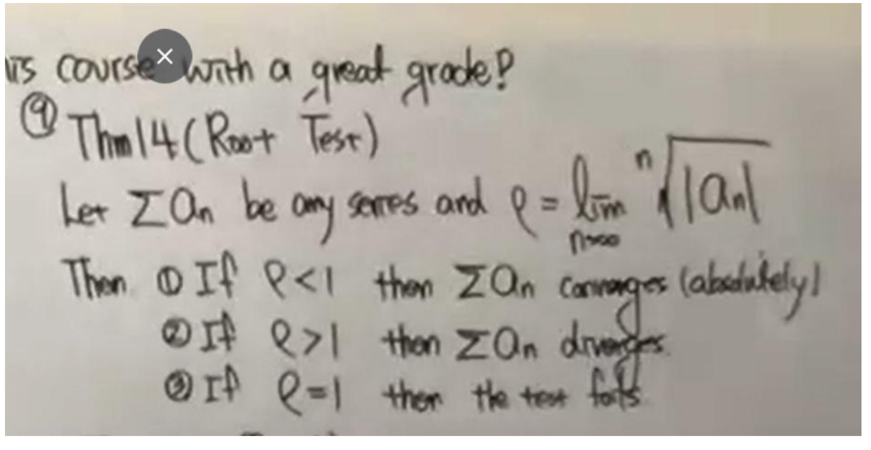
9. $\displaystyle\sum_{n=1}^{\infty} \frac{7}{(2n+5)^n}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{(3n)^n}$

11. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-5}\right)^n$

12. $\displaystyle\sum_{n=1}^{\infty} \left(-\ln\left(e^2+\frac{1}{n}\right)\right)^{n+1}$

11. $$\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{4n+3}{3n-5}\right)^n\right|} = \lim_{n\to\infty} \left(\frac{4n+3}{3n-5}\right) = \lim_{n\to\infty} \left(\frac{4}{3}\right) = \frac{4}{3} > 1 \Rightarrow \sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-5}\right)^n \text{ diverges}$$



Using the Root Test

In Exercises 9–16, use the Root Test to determine if each series converges absolutely or diverges.

9. $\displaystyle\sum_{n=1}^{\infty} \frac{7}{(2n+5)^n}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{(3n)^n}$

11. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-5}\right)^n$

12. $\displaystyle\sum_{n=1}^{\infty} \left(-\ln\left(e^2 + \frac{1}{n}\right)\right)^{n+1}$

$$12. \quad \lim_{n\to\infty} \sqrt[n]{\left|\left[-\ln\left(e^2+\frac{1}{n}\right)\right]^{n+1}\right|}$$

is course with a great grade?

9 Thm 14 (Root Test)

Let $\sum a_n$ be any series and $\rho = \lim_{n\to\infty} \sqrt[n]{|a_n|}$

Then ① If $\rho < 1$ then $\sum a_n$ converges (absolutely)

② If $\rho > 1$ then $\sum a_n$ diverges

③ If $\rho = 1$ then the test fails

Using the Root Test

In Exercises 9–16, use the Root Test to determine if each series converges absolutely or diverges.

9. $\displaystyle\sum_{n=1}^{\infty} \frac{7}{(2n+5)^n}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{(3n)^n}$

11. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-5}\right)^n$

12. $\displaystyle\sum_{n=1}^{\infty} \left(-\ln\left(e^2+\frac{1}{n}\right)\right)^{n+1}$

$$12. \quad \lim_{n\to\infty} \sqrt[n]{\left|\left[-\ln\left(e^2+\frac{1}{n}\right)\right]^{n+1}\right|} = \lim_{n\to\infty} \left[\ln\left(e^2+\frac{1}{n}\right)\right]^{1+1/n}$$

...is course with a great grade?

(9) Thm 14 (Root Test)

Let $\sum a_n$ be any series and $\rho = \lim_{n\to\infty} \sqrt[n]{|a_n|}$

Then ① If $\rho < 1$ then $\sum a_n$ converges (absolutely)

② If $\rho > 1$ then $\sum a_n$ diverges.

③ If $\rho = 1$ then the test fails.

Using the Root Test

In Exercises 9–16, use the Root Test to determine if each series converges absolutely or diverges.

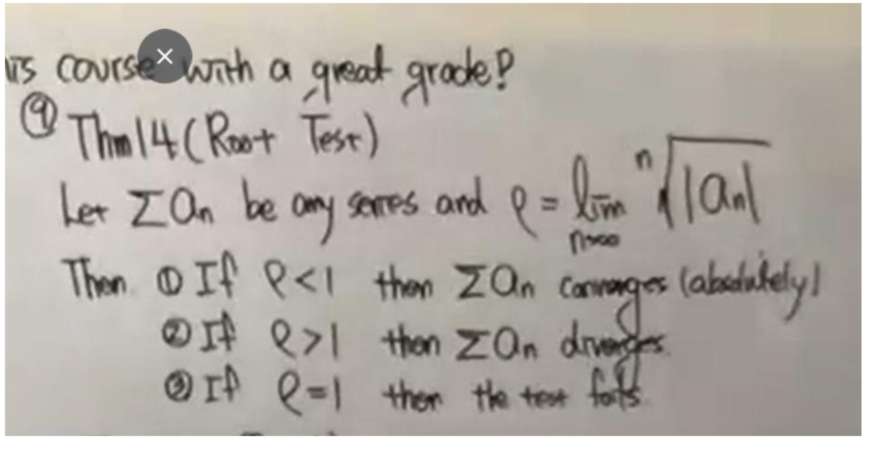
9. $\displaystyle\sum_{n=1}^{\infty} \frac{7}{(2n+5)^n}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{(3n)^n}$

11. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-5}\right)^n$

12. $\displaystyle\sum_{n=1}^{\infty} \left(-\ln\left(e^2+\frac{1}{n}\right)\right)^{n+1}$

$$12.\quad \lim_{n\to\infty} \sqrt[n]{\left|\left[-\ln\left(e^2+\frac{1}{n}\right)\right]^{n+1}\right|} = \lim_{n\to\infty}\left[\ln\left(e^2+\frac{1}{n}\right)\right]^{1+1/n} = \ln\left(e^2\right) = 2 > 1$$



Using the Root Test

In Exercises 9–16, use the Root Test to determine if each series converges absolutely or diverges.

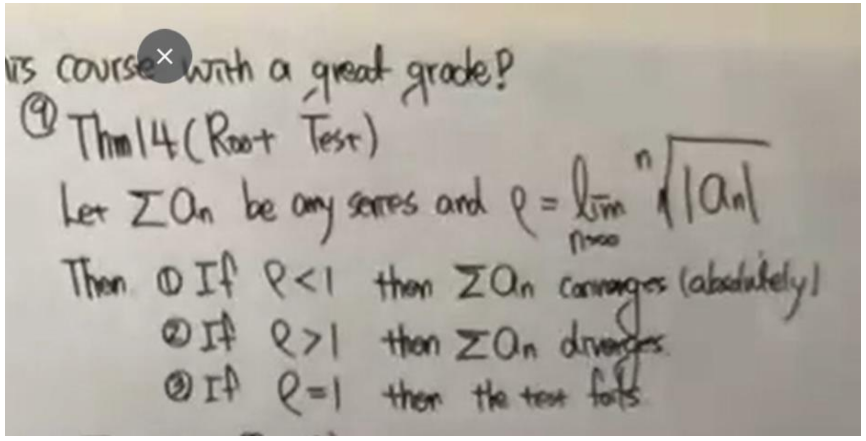
9. $\displaystyle\sum_{n=1}^{\infty} \frac{7}{(2n+5)^n}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{(3n)^n}$

11. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-5}\right)^n$

12. $\displaystyle\sum_{n=1}^{\infty} \left(-\ln\left(e^2+\frac{1}{n}\right)\right)^{n+1}$

$$12.\quad \lim_{n\to\infty} \sqrt[n]{\left|\left[-\ln\left(e^2+\frac{1}{n}\right)\right]^{n+1}\right|} = \lim_{n\to\infty}\left[\ln\left(e^2+\frac{1}{n}\right)\right]^{1+1/n} = \ln\left(e^2\right) = 2 > 1 \Rightarrow \sum_{n=1}^{\infty}\left[\ln\left(e^2+\frac{1}{n}\right)\right]^{n+1} \text{ diverges}$$

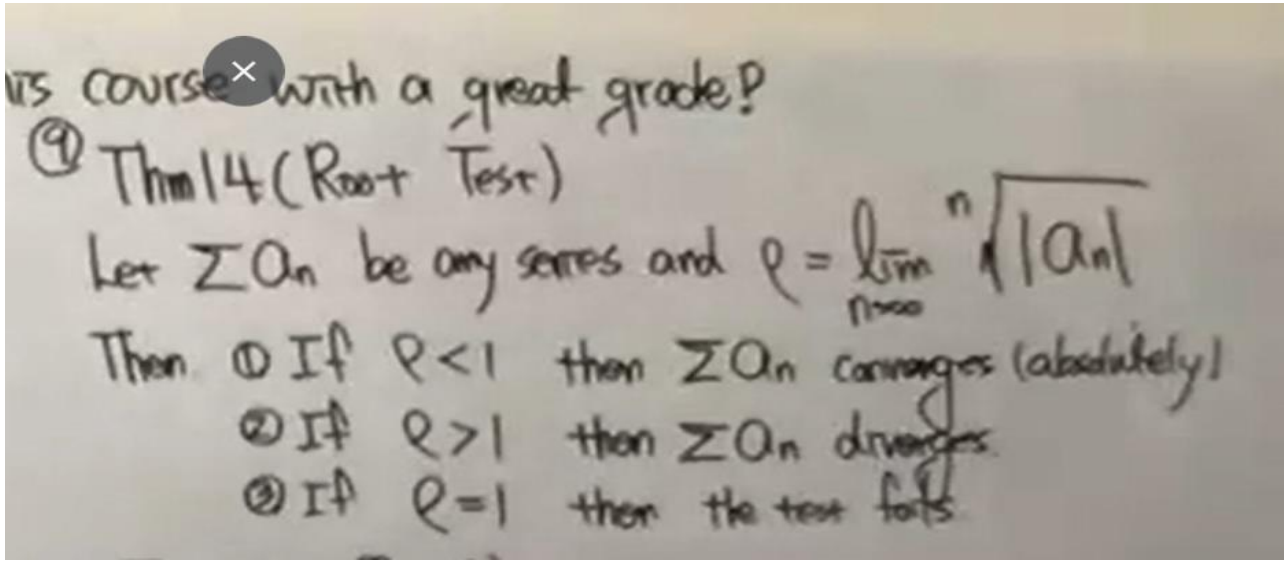


Any questions?

Homework - due next week .

Details will be announced – Canvas!

Stay safe!



Exam Advice:

Learn These! Learn These! Learn These! Learn These! Learn These!

... I will pass this course with a great grade!

Review (Test)

① Geometric Series Test $\sum ar^n$

$|r|<1$, then converge to $\frac{a_0}{1-r}$

$|r| \geq 1$, div

② P-Test $\sum \frac{1}{n^p}$ $p>1$ con, $p \leq 1$ div

③ Nth Term DT $\sum a_n$

$\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ div

④ Integral Test (i) Let $a_n = f(x)$ continuous, positive, decreasing

(ii) $\int_N^\infty f(x)dx$ con $\to \sum a_n$ con, div $\to \sum a_n$ div

⑤ DCT (BC → SC) (SD → BD)

⑥ LCT (i) $\lim_{n\to\infty} \frac{a_n}{b_n} = L$, both C or both D

(ii) $= 0$ & $\sum b_n$ con $\to \sum a_n$ con

(iii) $= \infty$ & $\sum b_n$ div $\to \sum a_n$ div

10.5 Absolute Convergence Test; Ratio & Root Test

Def If $\sum |a_n|$ converges, then $\sum a_n$ converges absolutely

⑦ Thm 12 (Absolute Convergence Test)

$\boxed{\text{If } \sum |a_n| \text{ converges, then } \sum a_n \text{ converges}}$

Ex1] $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \cdots$

$\sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{1}{n^2} \right| = \sum_{n=1}^{\infty} \frac{1}{n^2}$ con by P-Test with $p = 2 > 1$

Thus, I con by ACT