Interesting* Research topics

By: José L. Pabón, PhD

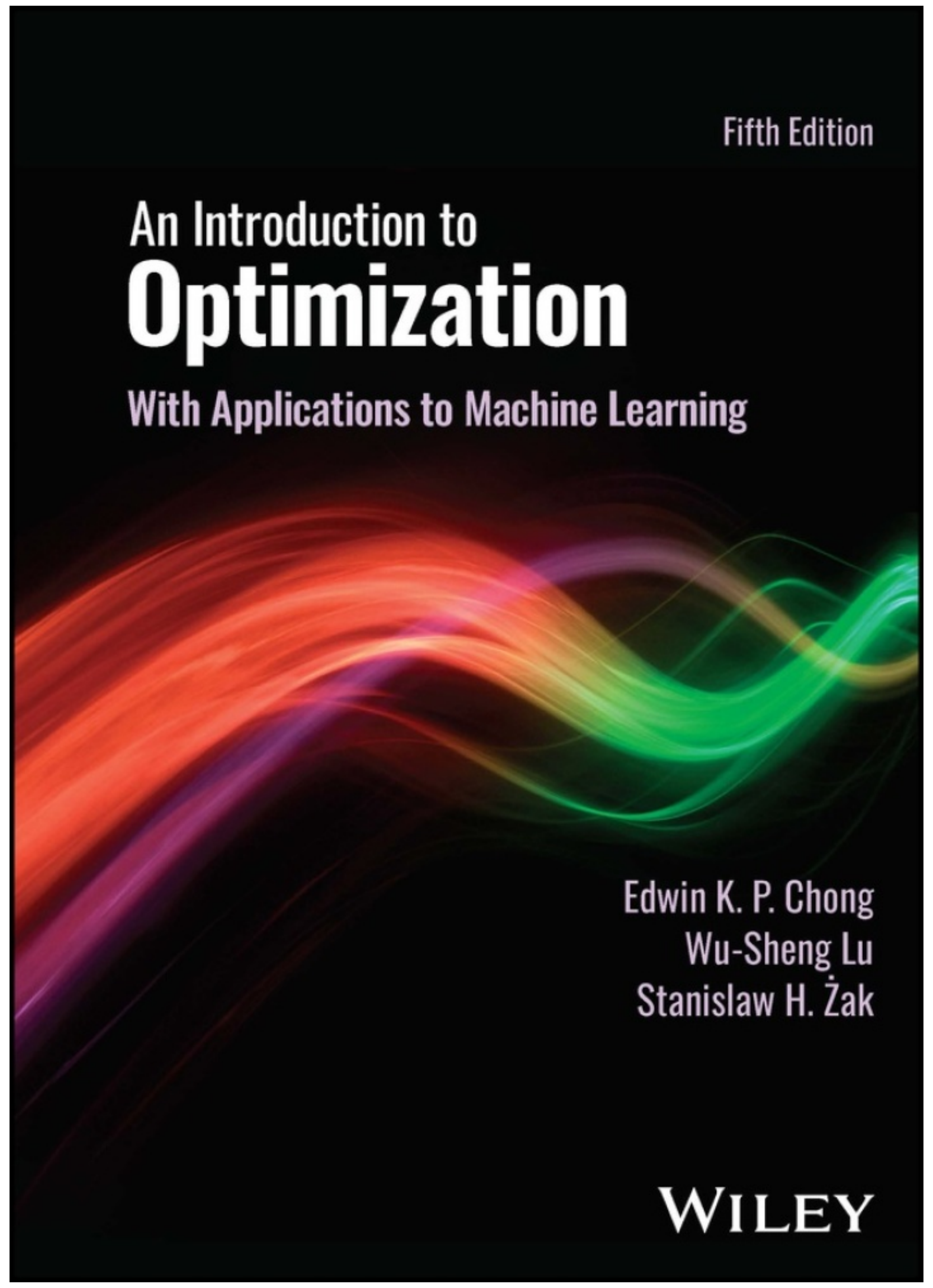
*** := I think they're interesting; students are encouraged to select other topics to be approved by Professor**

Research Topic - Optimization

- Mathematical optimization is a branch of math that is about finding a right set of answers that gives the best (optimal) solution to a problem, given some limitations, or criteria.
- In the simplest case, this means that an objective, often called an objective function needs to be minimized or maximized.

Research Topic - Optimization

- Mathematical optimization is a branch of math that is about finding a right set of answers that gives the best (optimal) solution to a problem, given some limitations, or criteria.
- In the simplest case, this means that an objective, often called an objective function needs to be minimized or maximized.



Research Topic - Optimization

- Mathematical optimization is a branch of math that is about finding a right set of answers that gives the best (optimal) solution to a problem, given some limitations, or criteria.
- In the simplest case, this means that an objective, often called an objective function needs to be minimized or maximized.

POKÉMON | FINANCE

Pokémon TCG: Chaos Rising Pull Rates

Making sense of Chaos.

By Peter Day | Published Jun 4, 2026 | 4 min read

The odds on specific cards of each rarity assume an equal population of each card at that rarity. The pull rates for each card we fo within the 95% confidence interval of an individual card of that rarity, which supports the assumption above.

Note: Your personal experience will vary—that's how randomized booster packs work!

Rarity	Pull Rate (95% Confidence Interval)	Rounded Pull Rate (Per X Packs)
Double Rare	**20.30%** ± 0.85%	**1 in 5** Packs
Ultra Rare	**8.29%** ± 0.58%	**1 in 12** Packs
Illustration Rare	**10.66%** ± 0.65%	**1 in 9** Packs
Special Illustration Rare	**1.21%** ± 0.23%	**1 in 83** Packs
Mega Hyper Rare	**0.10%** ± 0.07%	**1 in 956** Packs

Research Topic - Optimization

MAGIC: THE GATHERING | MODERN | COMPETITIVE

How Many Surveil Lands Should You Be Playing in Modern MTG?

Running the numbers on surveil 1.

By Frank Karsten | @karsten_frank | Updated Feb 13, 2025 | 11 min read

Number of surveil lands	Raw metagame share	Non-mirror winrate
0	53.9%	46.1%
1	19.5%	52.0%
2	22.4%	52.2%
3	2.4%	54.4%
4	1.9%	55.4%

MAGIC: THE GATHERING | MODERN | COMPETITIVE

How Many Surveil Lands Should You Be Playing in Modern MTG?

Running the numbers on surveil 1.

By Frank Karsten | @karsten_frank | Updated Feb 13, 20

Number of surveil lands	Raw metagame share	Non-mirror winrate
0	53.9%	46.1%
1	19.5%	52.0%
2	22.4%	52.2%
3	2.4%	54.4%
4	1.9%	55.4%

Surveil Lands in Dimir Mill

Market Price: **$2.83**

Market Price: **$3.40**

Out of all Dimir Mill players:

- 48.4% used 0 surveil lands, with a 66-67 (49.6%) record.
- 33.9% used 1 surveil land, with a 47-62 (43.1%) record.
- 17.7% used 2 surveil lands, with a 31-25 (55.4%) record.

MAGIC: THE GATHERING | MODERN | COMPETITIVE

How Many Surveil Lands Should You Be Playing in Modern MTG?

Running the numbers on surveil 1.

By Frank Karsten | @karsten_frank | Updated Feb 13, 20

Number of surveil lands	Raw metagame share	Non-mirror winrate
0	53.9%	46.1%
1	19.5%	52.0%
2	22.4%	52.2%
3	2.4%	54.4%
4	1.9%	55.4%

Surveil Lands in Dimir Mill

Market Price: **$2.83**

Market Price: **$3.40**

Out of all Dimir Mill players:

- 48.4% used 0 surveil lands, with a 66-67 (49.6%) record.
- 33.9% used 1 surveil land, with a 47-62 (43.1%) record.
- 17.7% used 2 surveil lands, with a 31-25 (55.4%) record.

- The article analyzed the win rates of nearly 6,000 competitive Modern decks from premier competitive tournaments in February and March of year 2025.
- In general, one surveil land is the sweet spot for most fair multicolor Modern decks, and a second surveil land can be worth it for combo strategies.
- Archetypes that rely on graveyards or cascade spells, such as non-DomRhinos, Living End or Goryo's Vengeance, may want to run three or four, but you need good reasons to do so.
- Overall, surveil lands were a major addition to the format, and players who adopted them quickly reaped the rewards.

The Math of **Meaning**

How Markov Chains and Probability Theory scaffold the AI that writes like Shakespeare and Taylor Swift.

Probability

- Probability or Stochastics is a part of applied mathematics.It has to do with chance, the study of things that might happen or might not happen.

- A Stochastic process is a mathematical description of random events that occur one after another. It is possible to order these events according to the time at which they occur.
- This can be used to model such things as daily weather data, or exchange rate changes, or medical information like a patient's EKG, EEG, blood pressure or temperature.
- Stochastic processes used in various disciplines, including physics, biology, finance, telecommunications, and operations research. They provide a powerful framework for analyzing and predicting the behavior of systems under uncertain conditions.
- Combinatorics is a branch of mathematics. It is concerned with finite or countable infinite sets. Combinatorics is part of discrete mathematics.
- Combinatorics looks at counting the number of possibilities, and about the questions whether certain configurations exist, and how to get to them.

Neural Networks

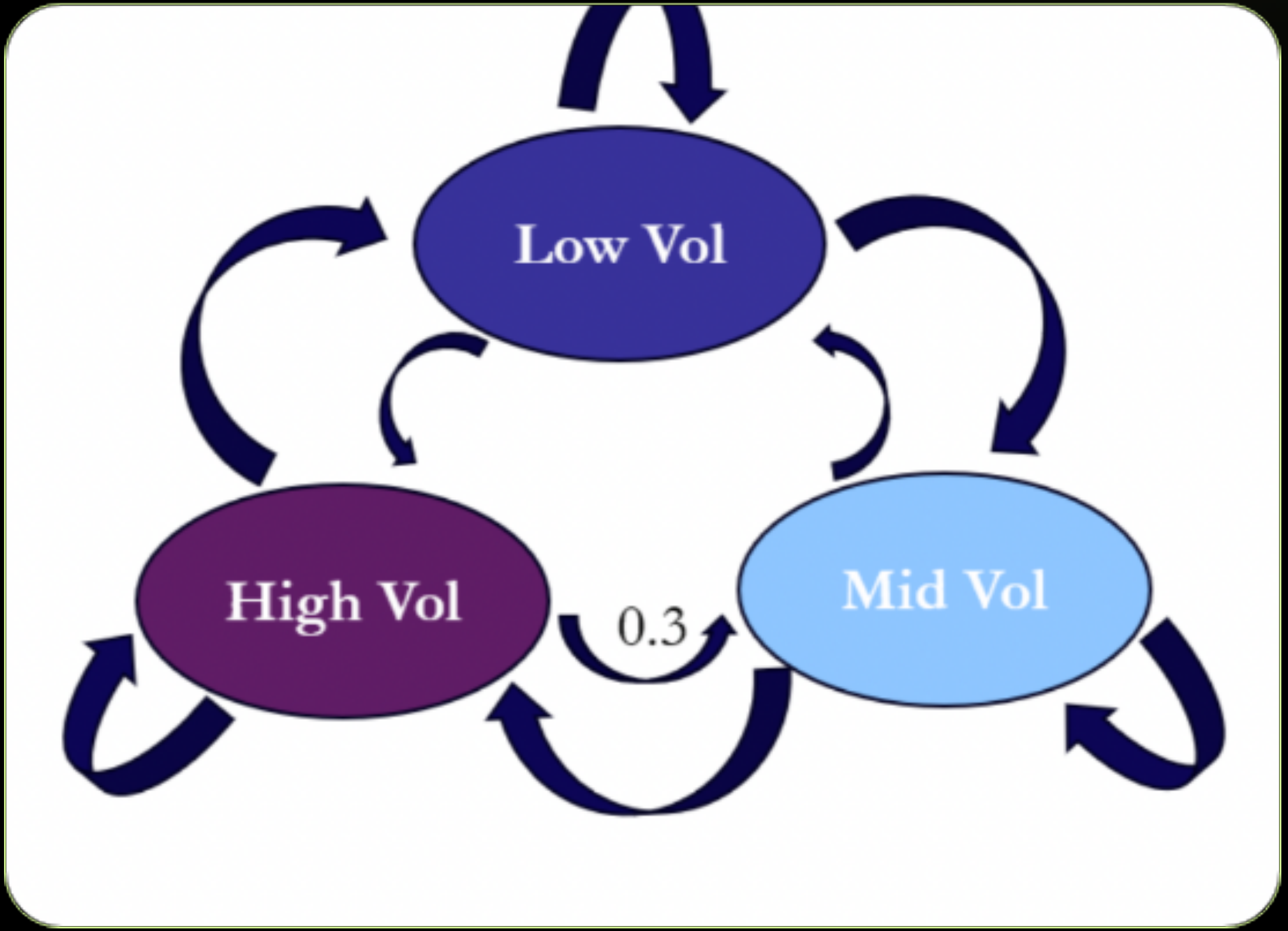
- A (biological) neuron (or neurone) is a nerve cell which uses electrical impulses to carry out their purpose. Neurons are the basic units of our nervous system. Their purpose in the nervous system is to receive, transmit, and carry electrical impulses called action potentials.
- A group of connected neurons is called a neural network.
- A neural network (also called an ANN or an artificial neural network) is a sort of computer software, inspired by biological neurons.
- Biological brains are capable of solving difficult problems, but each neuron is only responsible for solving a very small part of the problem.
- A neural network is made up of cells that work together to produce a desired result, although each individual cell is only responsible for solving a small part of the problem.
- This is one method for creating artificially intelligent programs.
- Neural networks are an example of machine learning, where a program can change as it learns to solve a problem.
- A neural network can be trained and improved with each example, but the larger the neural network, the more examples it needs to perform well—often needing millions or billions of examples in the case of deep learning.

The Markov Foundation

At its simplest level, AI text generation is a Markov Process. It assumes that the probability of the next word depends only on the current state.

$$PX_{n+1} \mid X_n$$

If the current word is "Love", a Taylor Swift-trained model might predict "Story" with 80% probability, while a Shakespeare model might predict "is".

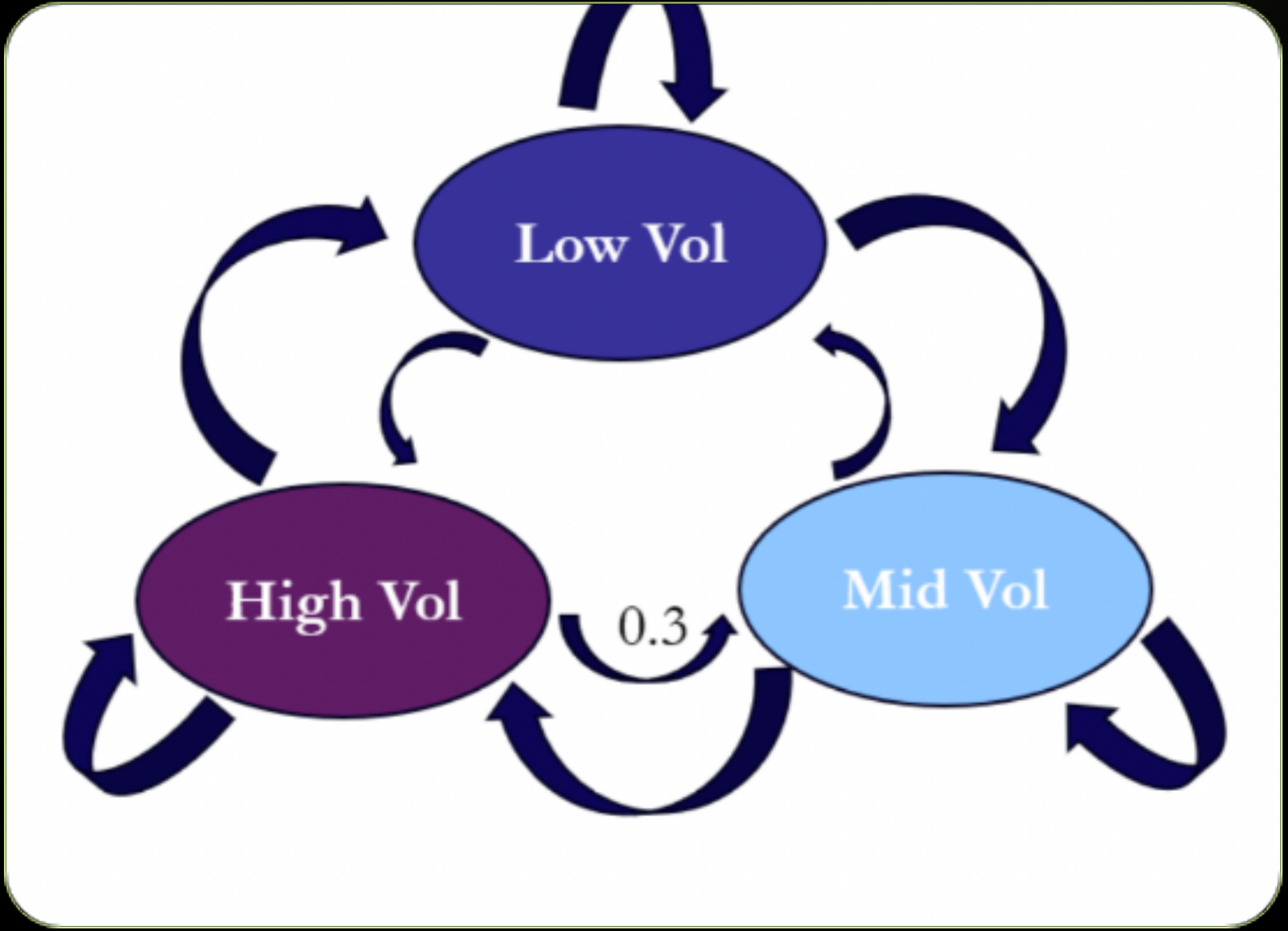


The Markov Foundation

At its simplest level, AI text generation is a Markov Process. It assumes that the probability of the next word depends only on the current state.

$$PX_{n+1} \mid X_n$$

A Markov chain is a model of some random process that happens over time. Markov chains are called that because they follow a rule called the Markov property. The Markov property says that whatever happens next in a process only depends on how it is right now (the state). It doesn't have a "memory" of how it was before. It is helpful to think of a Markov chain as evolving through discrete steps in time, although the "step" doesn't need to have anything to do with time.



Scaffolding with Probability

Tokenization

AI doesn't read words; it reads numbers. Shakespeare's "Thou" becomes a unique ID in a multi-dimensional matrix.

Distributions

Training is the process of counting. We build a massive frequency table of which "tokens" usually follow others in Taylor's discography.

Stochasticity

The AI doesn't always pick the most likely word (that's boring). It "samples" from the distribution to create variety and creativity.

From Chains to Neural Nets

A basic Markov Chain only looks at the current word. Modern AI (Neural Networks) uses millions of "weights" to remember the entire context.

This allows the model to maintain the "vibe" of a Taylor Swift bridge or the structure of a Shakespearean sonnet across hundreds of words.

Contextual Weight Calculation:

$$\text{Attention}(Q, K, V) = \text{softmax}\frac{QK^T}{\sqrt{d_k}}V$$

The Result: Synthetic Soul

Bard Mode

"Shall I compare thee to a summer's day? / Thou art more lovely and more temperate..."

The model identifies iambic pentameter as a high-probability structure.

Swiftie Mode

"It's a love story, baby, just say yes / I'm standing there, on a balcony in summer air..."

The model recognizes lyrical motifs of heartbreak, memory, and vivid imagery.

AI doesn't "know" Shakespeare or Taylor—it just knows the statistical path they carved through language.

“Direction”

Professor will do quick demonstration on the board.

“Direction”

Professor will do quick demonstration on the board.

A machine learning model, trains upon a large set (like a dictionary)…

Then if you ask it ‘man is to woman as King is to?’

(More often that not)

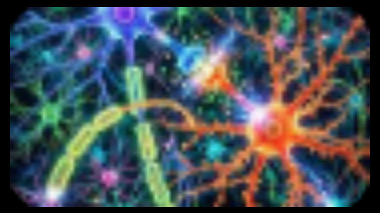
It will answer:

Queen.

Image Sources

https://bsic.it/wp-content/uploads/2025/11/Screenshot-2025-11-02-at-14.01.32.png

Source: bsic.it



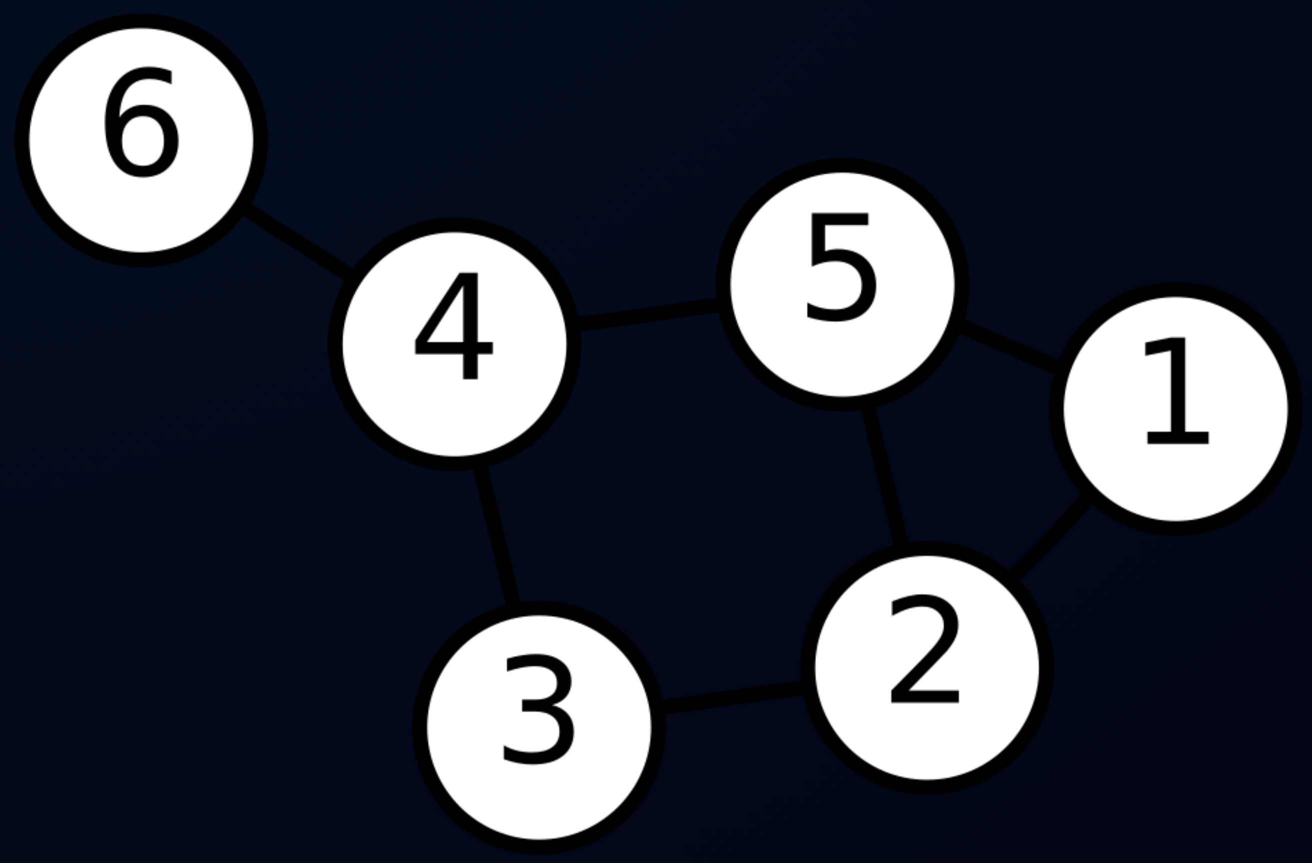
https://media.easy-peasy.ai/8f0dd6c2-432a-4079-8cf0-66c2498f9360/8ffc5c95-cf8c-463e-b3dc-7ec7a1b178ce.png

Source: easy-peasy.ai

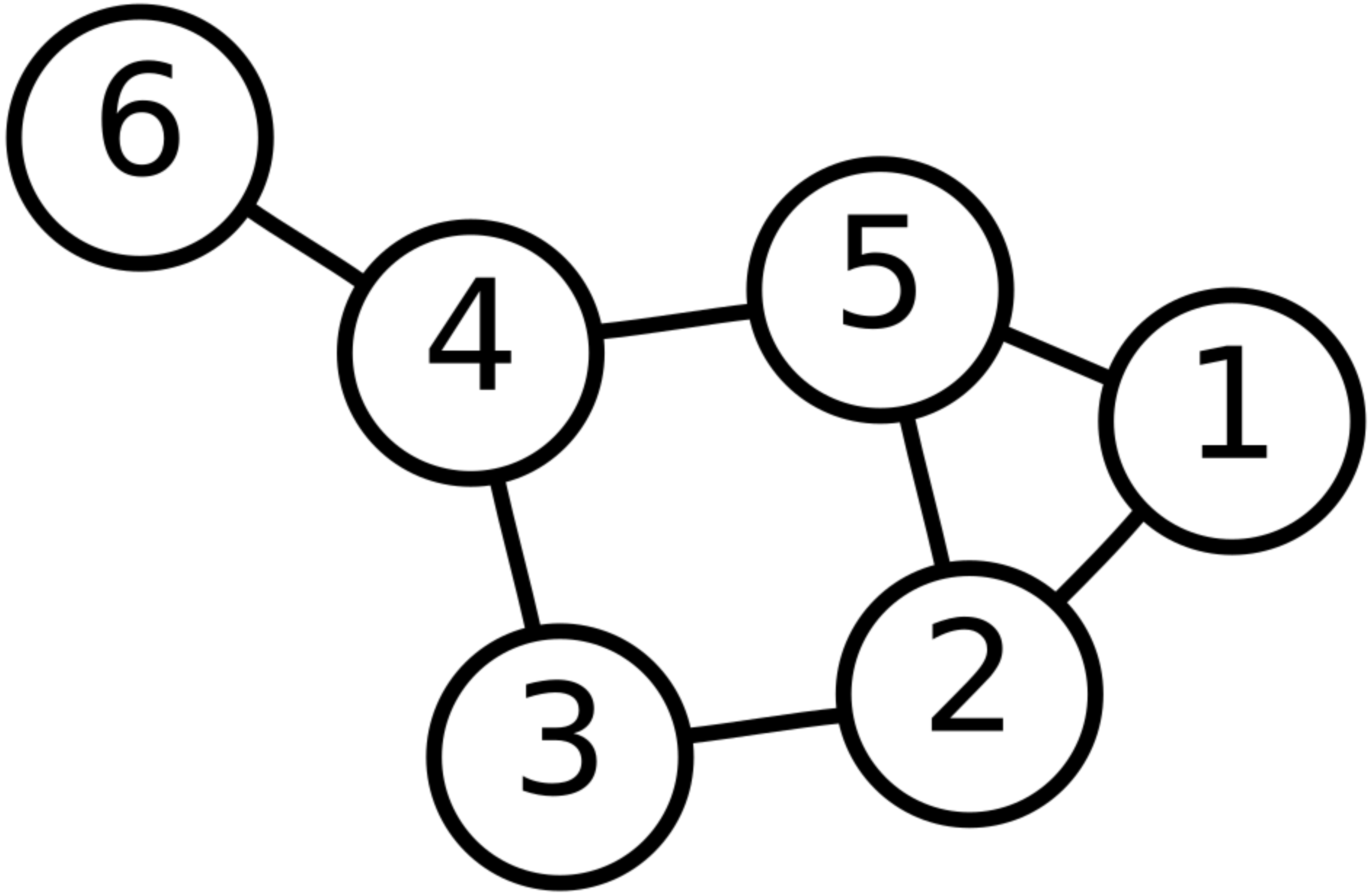
The Power of Graph Theory

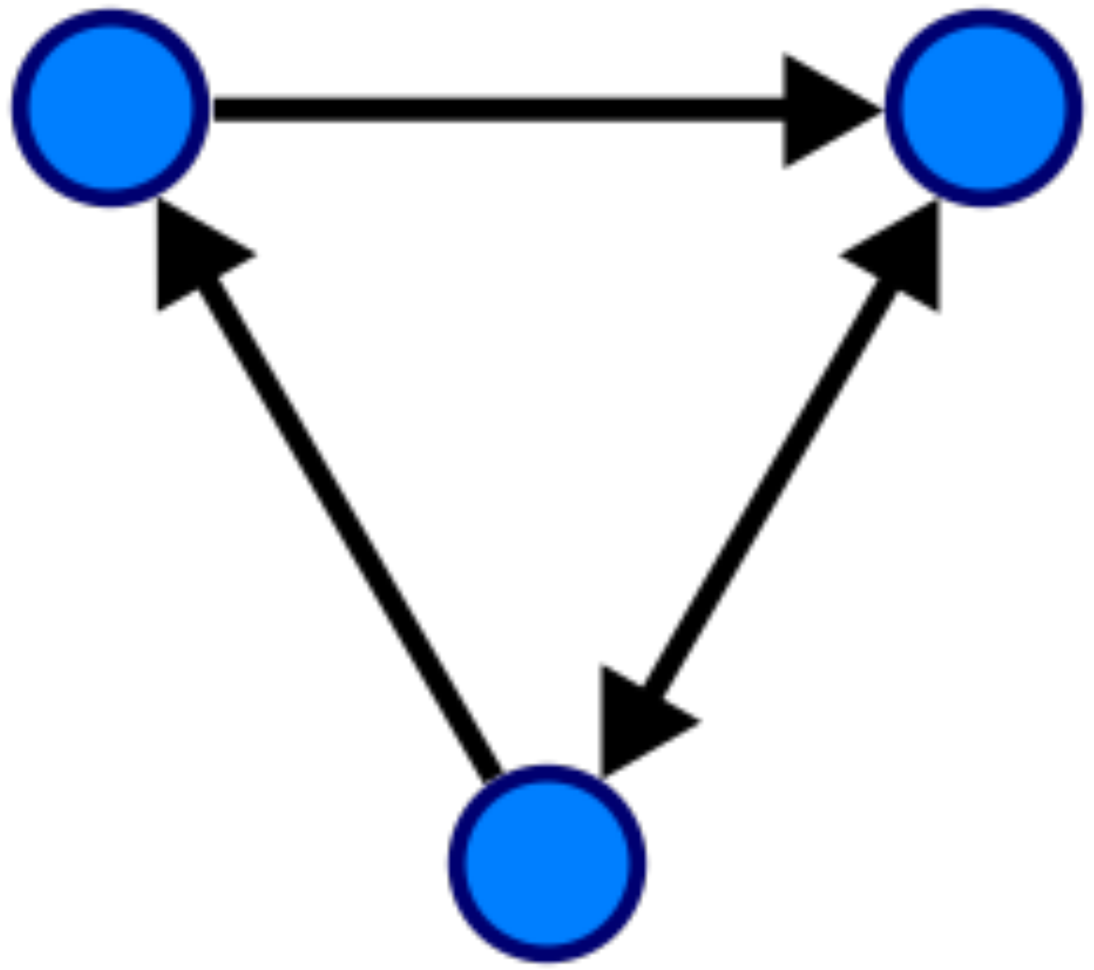
From Netflix Recommendations to Google Search: How modern platforms model the world through connections.

Graph Theory

- Graph theory is a field of mathematics about graphs. A graph is an abstract representation of: a number of points that are connected by lines. Each point is usually called a vertex (more than one are called vertices), and the lines are called edges. Graphs are a tool for modelling relationships. They are used to find answers to a numbe of problems.
- 

Graph Theory





Graph Theory

- Graph theory is a field of mathematics about graphs. A graph is an abstract representation of: a number of points that are connecte by lines. Each point is usually called a vertex (more than one are called vertices), and the lines are called edges. Graphs are a tool f modelling relationships. They are used to find answers to a number of problems.

- Some of these questions are:

- What is the best way for a mailman to get to all of the houses in the area in the least amount of time? The points could represent street corners and lines could represent the houses along the street. (see Chinese postman problem)

- A salesman has to visit different customers, but wants to keep the distance traveled as small as possible. The problem is to find a way so they can do it. This problem is known as Travelling Salesman Problem (and often abbreviated TSP). It is among the hardest problems to solve. If a commonly believed conjecture is true (described as $P \neq NP$), then an exact solution requires one to try all possible routes to find which is shortest.

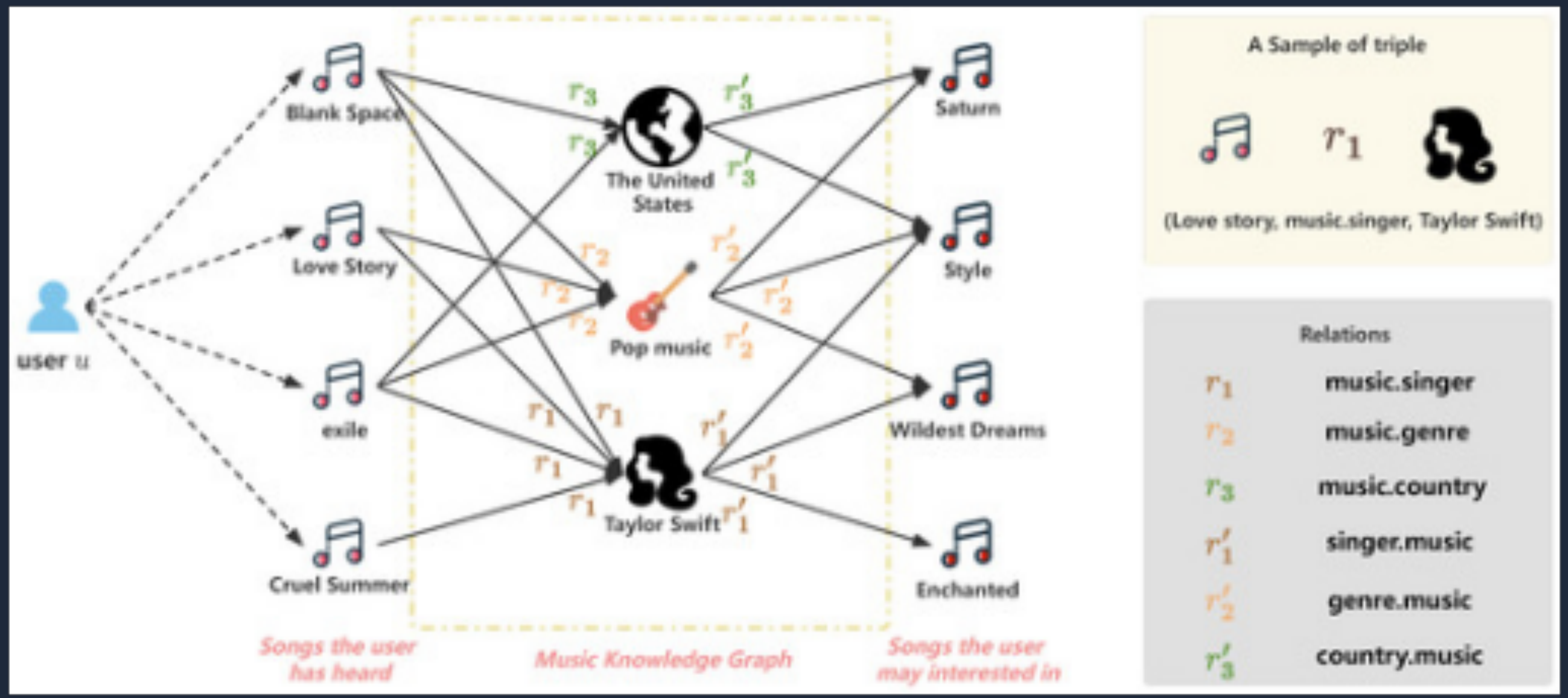
- How many colors would be needed to color a map, if countries sharing a border are colored differently? The points could represen the different areas and the lines could represent that two areas are neighboring. (look at the Four color theorem)

- Can a sketch be drawn in one closed line? The lines of the drawing are the lines of the graph and when two or more lines collide, there is a point in the graph. The task is now to find a way through the graph using each line one time. (look at Seven Bridges of Königsberg)

MAPPING YOUR TASTE

Platforms like Netflix use Bipartite Graphs to store data. In these graphs:

- Nodes represent two distinct sets: Users and Movies.
- Edges exist only between a user and a movie (a watch or a rating).
- By finding users with similar "connection patterns," the graph predicts new edges—recommending your next favorite show.



THE ALGORITHM OF IMPORTANCE

Google’s PageRank treats the web as a directed graph. Each page is a node, and every link is a "vote" of confidence.

- A page's rank depends on the rank of the pages linking to it.
- Links from high-authority sites carry more weight.
- The Damping Factor (d) accounts for the "Random Surfer" who might jump to any random page.

$$PR(A) = \frac{1 - d}{N} + d \sum_{v \in B_A} \frac{PR(v)}{L(v)}$$

Where PR(A) is the rank, N is the total pages, and L(v) is the number of outbound links.

The Graph is Everything

Whether finding a route home or finding your next movie, graph theory provides the structure for modern intelligence.

THANK YOU FOR YOUR ATTENTION, Questions?

IMAGE SOURCES

https://www.mdpi.com/electronics/electronics-12-02688/article_deploy/html/images/electronics-12-02688-g001-550.jpg

Source: www.mdpi.com

https://png.pngtree.com/thumb_back/fw800/background/20251204/pngtree-abstract-network-of-glowing-nodes-and-connections-on-a-dark-blue-image_20733426.webp

Source: pngtree.com

Graph Theory

- Questions?
-

Graph Theory

- Graph theory is a field of mathematics about graphs. A graph is an abstract representation of: a number of points that are connected by lines. Each point is usually called a vertex (more than one are called vertices), and the lines are called edges. Graphs are a tool for modelling relationships. They are used to find answers to a number of problems.
- Some of these questions are:
- What is the best way for a mailman to get to all of the houses in the area in the least amount of time? The points could represent street corners and lines could represent the houses along the street. (see Chinese postman problem)
- A salesman has to visit different customers, but wants to keep the distance traveled as small as possible. The problem is to find a way so they can do it. This problem is known as Travelling Salesman Problem (and often abbreviated TSP). It is among the hardest problems to solve. If a commonly believed conjecture is true (described as $P \neq NP$), then an exact solution requires one to try all possible routes to find which is shortest.
- How many colors would be needed to color a map, if countries sharing a border are colored differently? The points could represent the different areas and the lines could represent that two areas are neighboring. (look at the Four color theorem)
- Can a sketch be drawn in one closed line? The lines of the drawing are the lines of the graph and when two or more lines collide, there is a point in the graph. The task is now to find a way through the graph using each line one time. (look at Seven Bridges of Königsberg)
-

Modeling

- A **mathematical or physics model** can be a description of a physical system using mathematical concepts and language.
- The process of building a mathematical model is termed mathematical modeling.
- Mathematical models are used in the natural sciences (such as physics, biology, earth science, meteorology) and engineering disciplines (e.g. computer science, artificial intelligence).
- They are also used in the social sciences (such as economics, psychology, sociology and political science).
- Physicists, engineers, statisticians, operations research analysts and economists use mathematical models a lot.

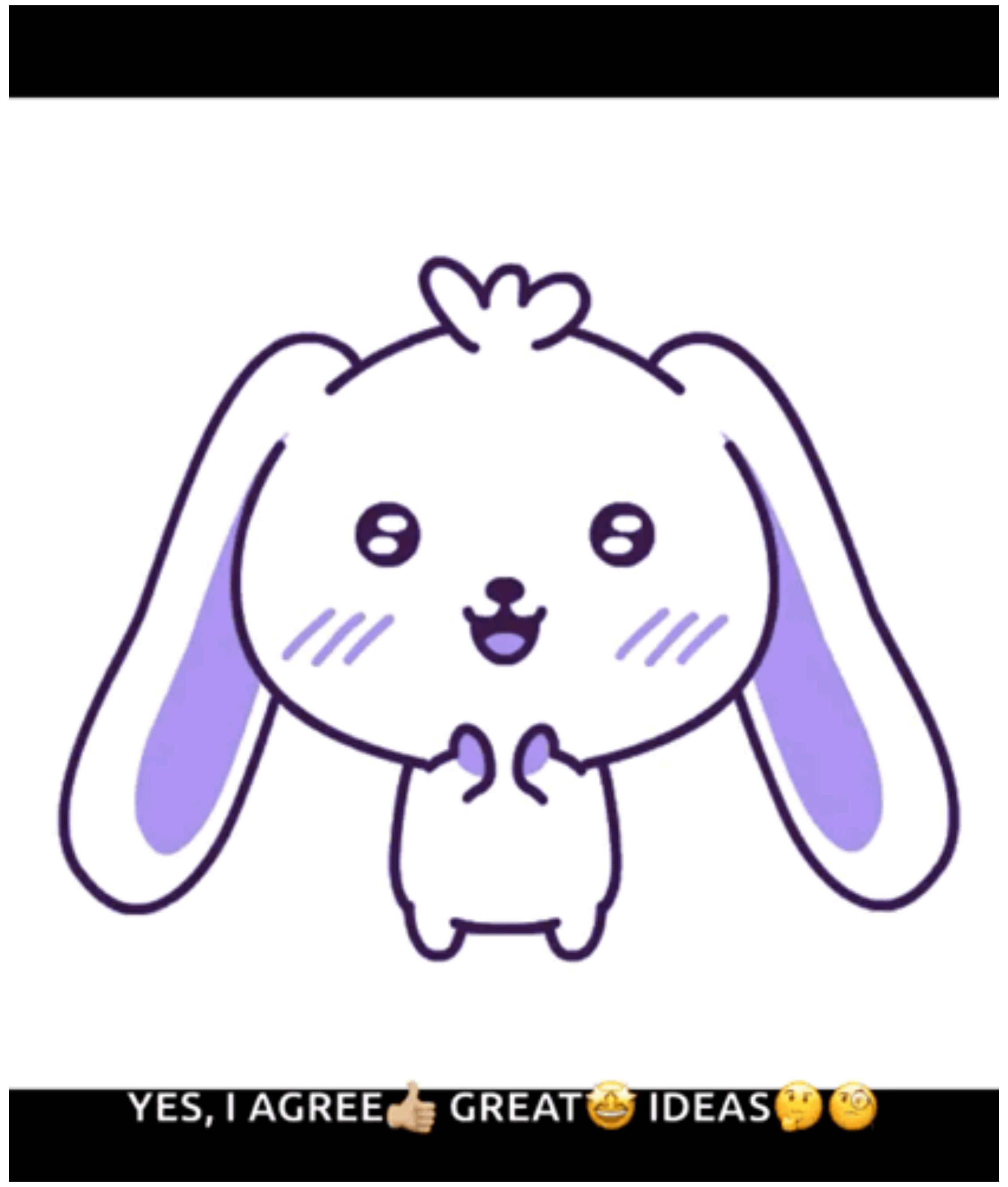
Dynamical system

- A Dynamical system is a mathematical or physics model of a process, that is dependent on time.
- The process depends on the initial state, typically.
- The current terminology was established by Henri Poincaré and George David Birkhoff.
- Dynamical systems are very common.
- Examples are the movement of a pendulum in physics, or the predator-prey equations in biology.

Dynamical systems modeling

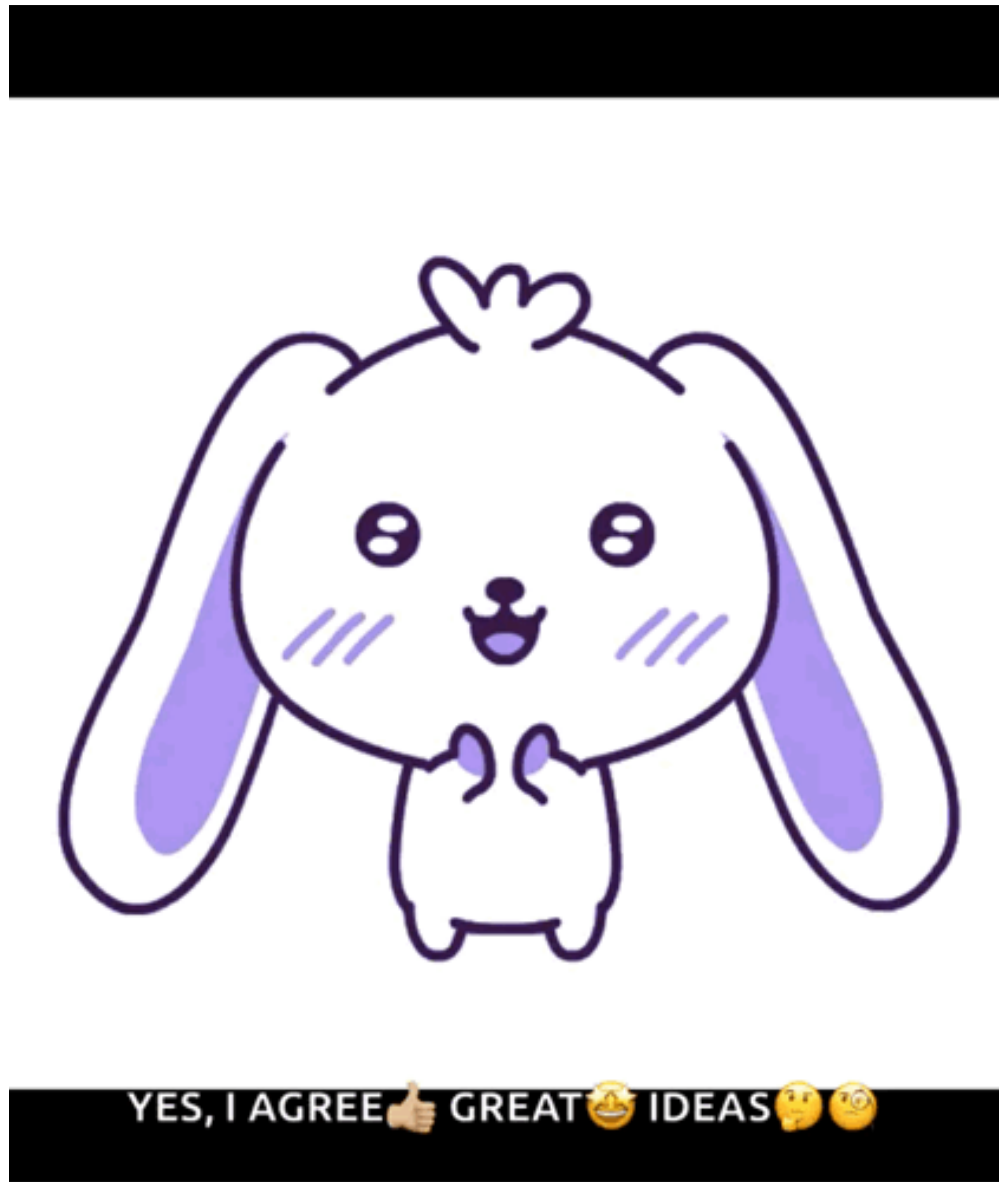
- All models are wrong…
- But some are useful.

Dynamical systems modeling



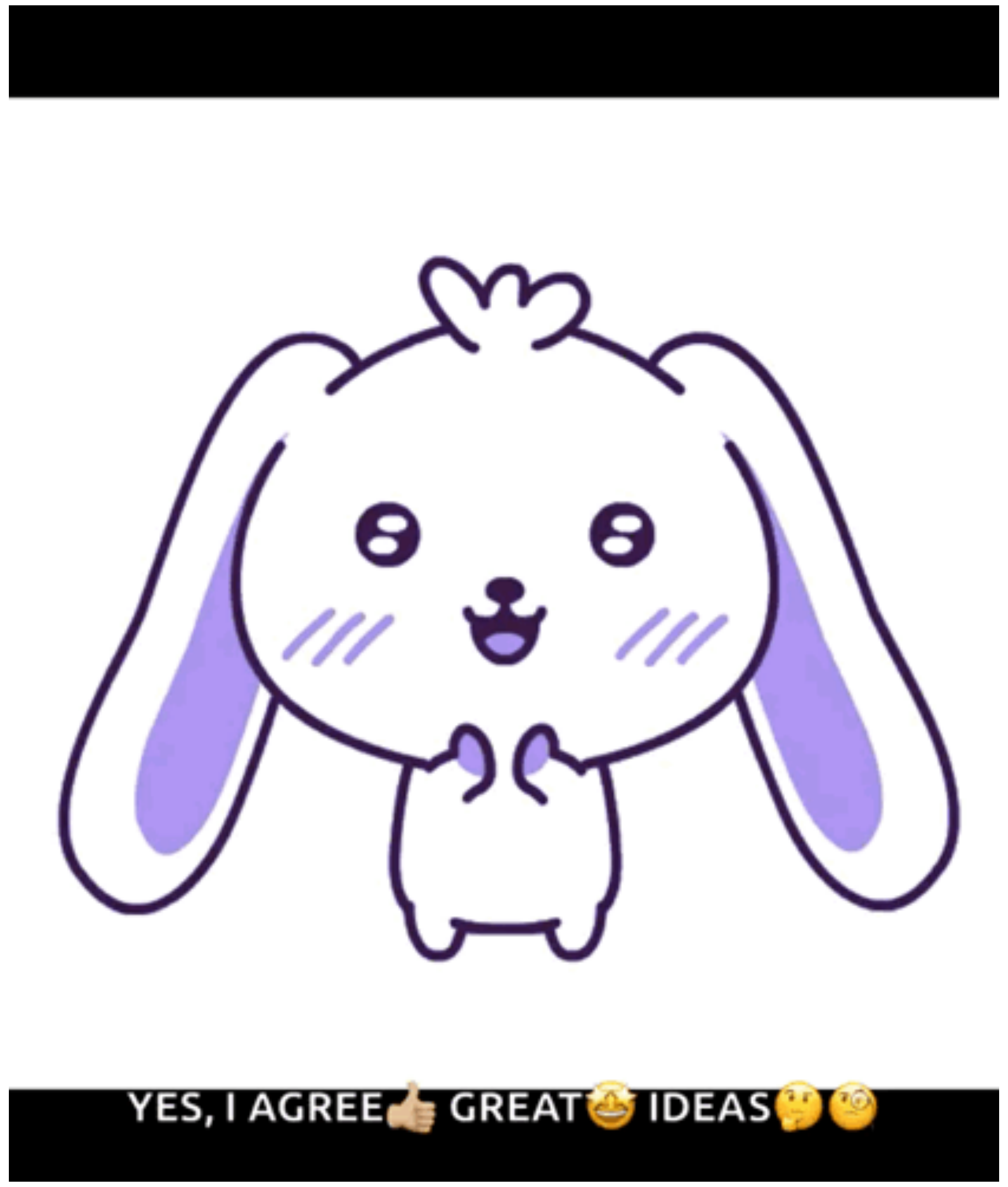
- A model of… bunny populations!
- This model is well known - a logistic map.

Dynamical systems modeling



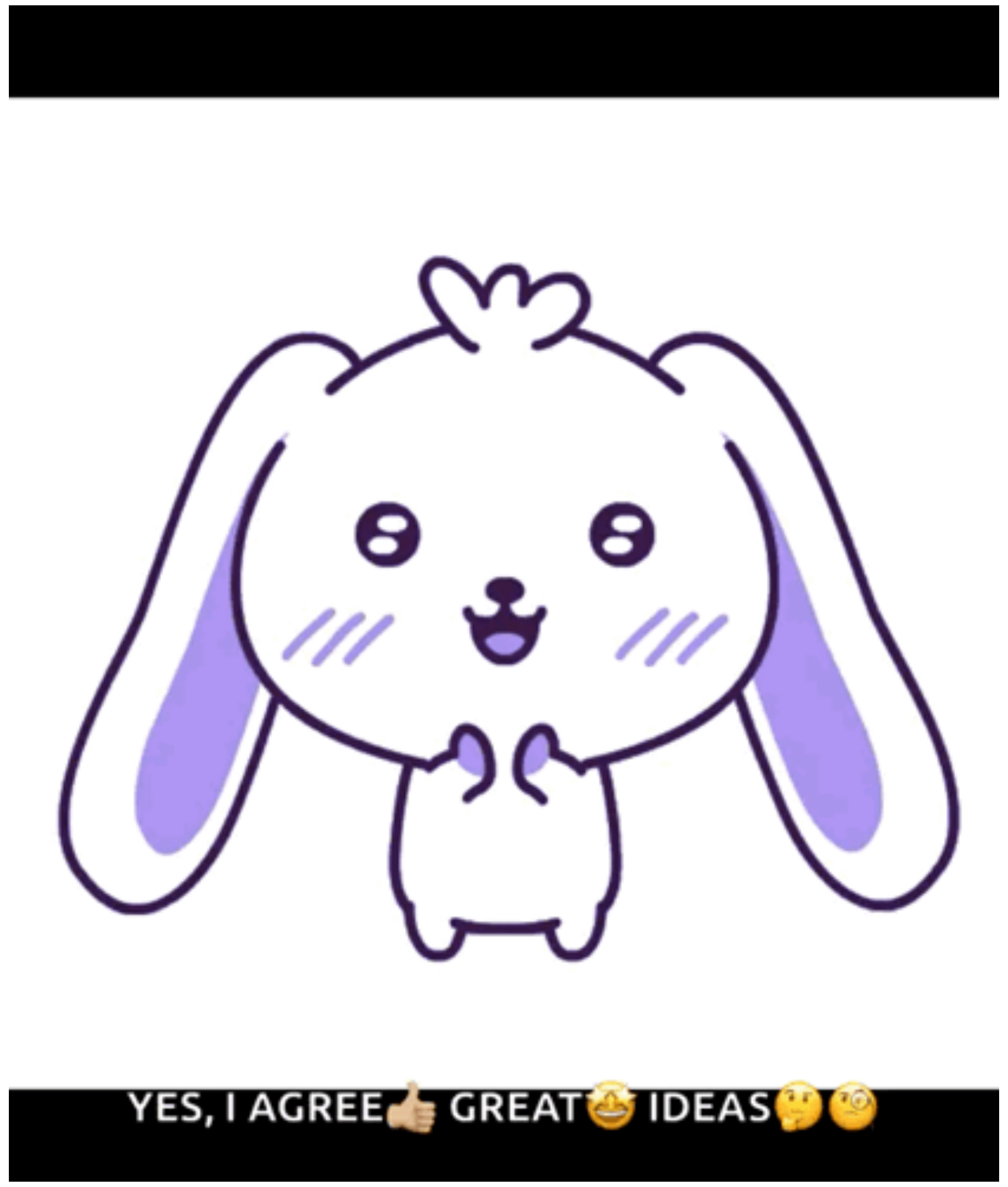
- A model of… bunny populations!
- This model is well known - a logistic map.
- Let X = population of bunnies.
- r = rate at which they reproduce or grow.
- How does the population grow over time?
- Let time be discrete, starting at t=0, then t=1,2,3, …
- $X_{current+1} = rX_{current}$

Dynamical systems modeling



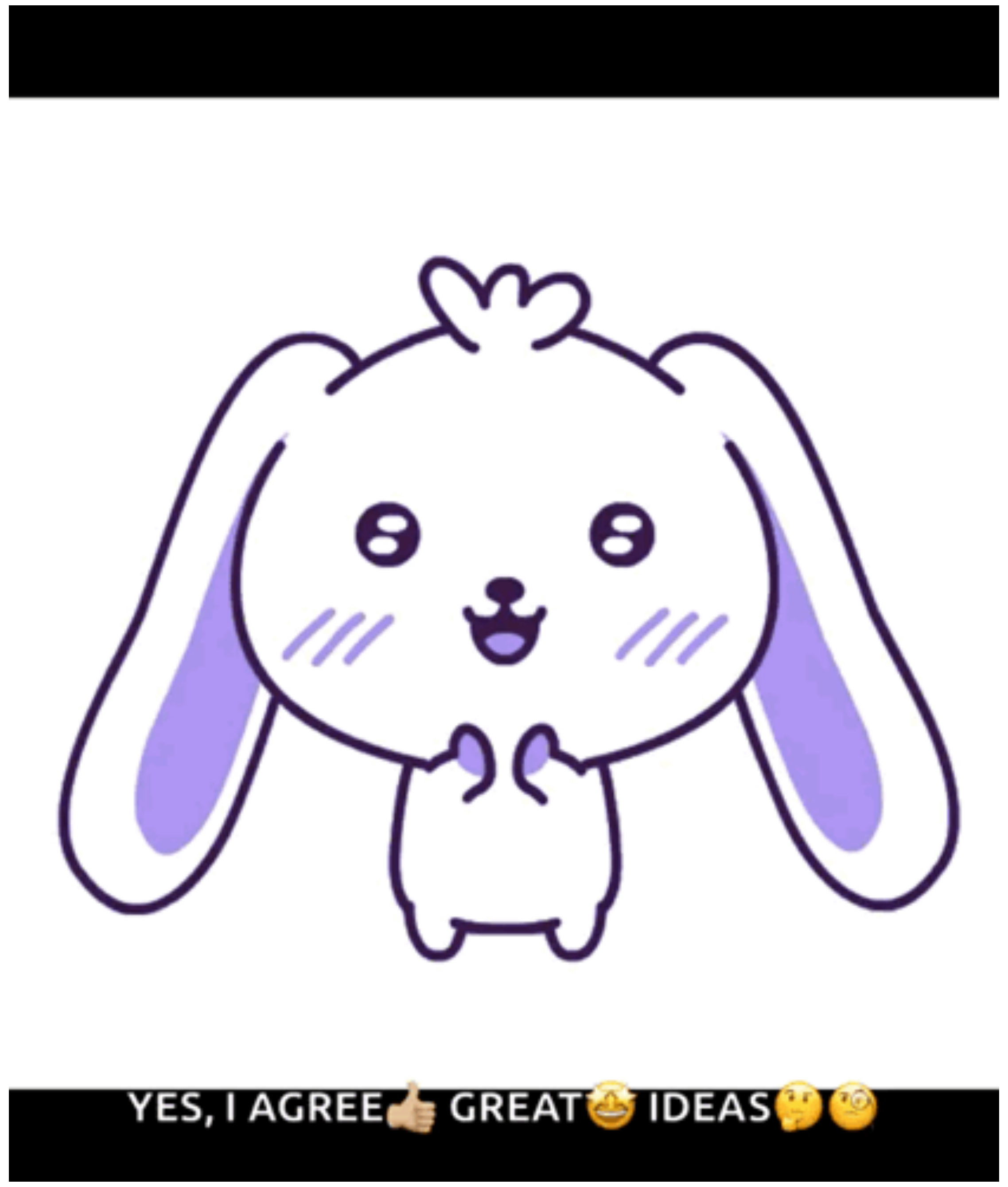
- A model of… bunny populations!
- This model is well known - a logistic map.
- Let X = population of bunnies.
- r = rate at which they reproduce or grow.
- How does the population grow over time?
- Let time be discrete, starting at t=0, then t=1,2,3, …
- $X_{current+1} = rX_{current}$
- Strengths of this model? Weakness of this model?

Dynamical systems modeling



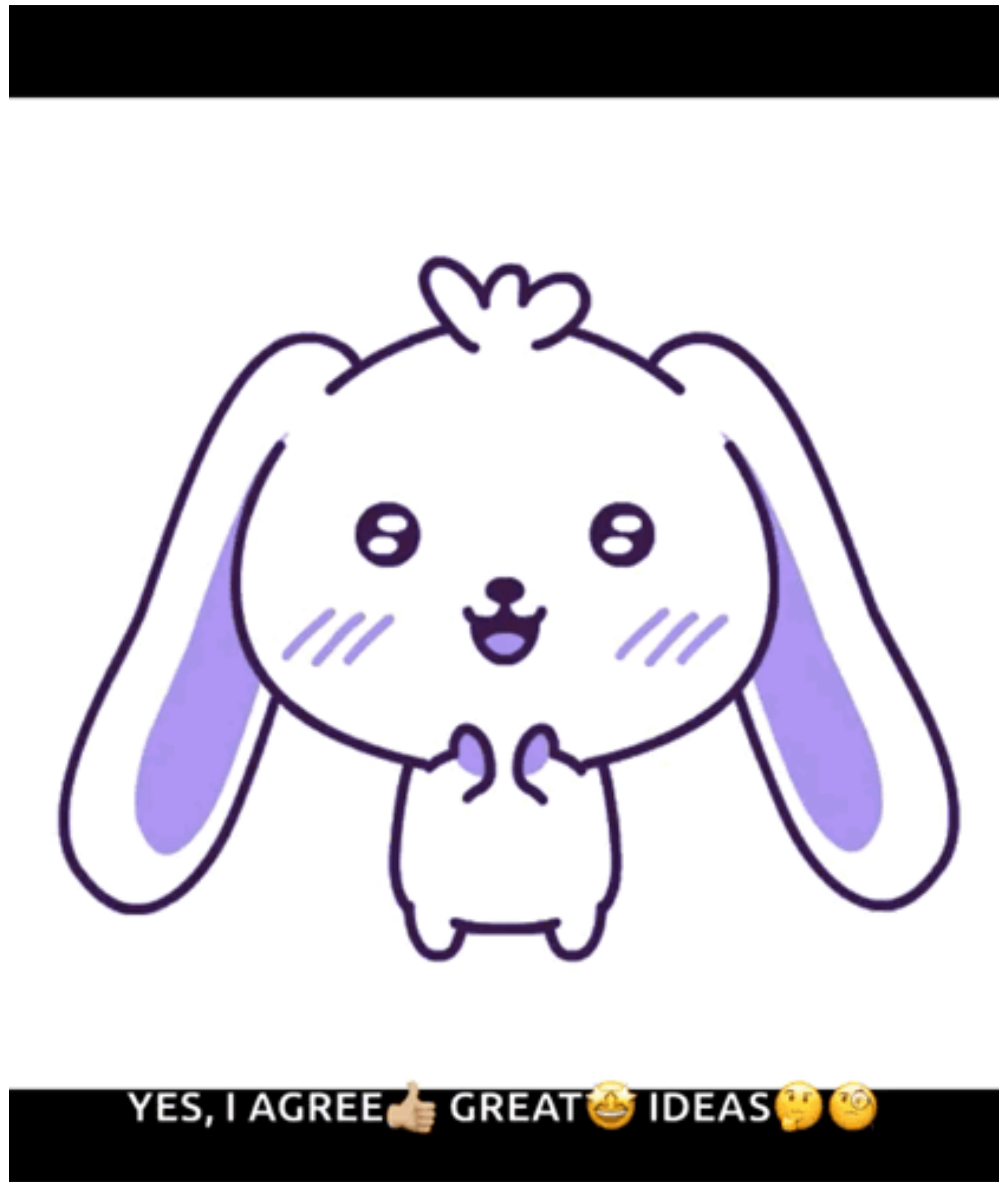
- Another model of bunny population- a logistic map.
- Let X = population of bunnies.
- r = rate at which they reproduce or grow.
- Let time be discrete, starting at t=0, then t=1,2,3, …
- $X_{current+1} = rX_{current}\left(1 - X_{current}\right)$
- Strengths of this model? Weakness of this model?

Dynamical systems modeling



- Let X = population of bunnies.
- r = rate at which they reproduce or grow.
- $X_{current+1} = rX_{current}\left(1 - X_{current}\right)$
- Note - the initial amount of bunnies, known as initial condition, does not affect the final population.
- *r* is known as the bifurcation parameter.
- For low *r*, the bunny population goes extinct.
- For *r* values up to ~2.49, you get convergence to a value.

Dynamical systems modeling



- Let X = population of bunnies.
- r = rate at which they reproduce or grow.
- $X_{current+1} = rX_{current}\left(1 - X_{current}\right)$
- Note - the initial amount of bunnies, known as initial condition, does not affect the final population.
- *r* is known as the bifurcation parameter.
- For *r* values between 2.49 and 4… periodicity… and chaos!
- For higher *r* values, the system radically changes: the system diverges and goes to negative infinity for almost all initial starting conditions.

PHYSICS OF THE WORLD CUP

José L. Pabón PhD

Conservation of Momentum and Uniform Motion in Soccer

DEFINING LINEAR MOMENTUM

OPENSTAX SECTION 8.1

Momentum is a vector quantity that describes an object's "quantity of motion." It depends on both the object's mass and its velocity.

$$p = m \cdot v$$

- **Mass (m):** A standard soccer ball is ~0.45 kg.
- **Velocity (v):** Top shots exceed 30 m/s.

FORCE AND IMPULSE

OPENSTAX SECTION 8.2

Newton's Second Law can be restated in terms of momentum: the net force equals the rate of change of momentum.

$$F_{net} = \frac{\Delta p}{\Delta t}$$

Impulse (J) is the product of force and time, resulting in a change in momentum.

Goalkeepers use "soft hands" to increase contact time and reduce peak impact force.

THE CONSERVATION LAW

OPENSTAX SECTION 8.3

In an isolated system (where the net external force is zero), the total momentum remains constant.

$$\Sigma p_{initial} = \Sigma p_{final}$$

During a collision between a player's foot and the ball, momentum is transferred. The sum of the foot's momentum and the ball's momentum before the strike equals the sum after impact.

MOMENTUM & UNIFORM MOTION

Uniform motion occurs when an object travels in a straight line at a constant speed (zero acceleration).

THE MOMENTUM PERSPECTIVE

If $F_{net} = 0$, then:

- → Change in momentum (Δp) is zero.
- ✓ Momentum is conserved (p is constant).
- = Velocity remains constant (Uniform Motion).

KEY INSIGHT

Once a soccer ball is kicked and leaves the foot (ignoring air resistance and friction), it enters a state of **uniform motion** in the horizontal direction. Its momentum remains constant because no horizontal forces are acting on it.

SESSION SUMMARY

- **Momentum:** Mass in motion ($p = mv$).
- **Impulse:** Force over time creates Δp.
- **Conservation:** Total momentum is constant in isolated systems.

?

Any Questions?

IMAGE SOURCES

https://png.pngtree.com/png-vector/20260323/ourlarge/pngtree-dynamic-soccer-ball-flying-fast-with-motion-blur-effect-on-white-png-image_18689495.webp

Source: pngtree.com

https://png.pngtree.com/thumb_back/fw800/background/20250913/pngtree-goalkeeper-makes-dramatic-diving-save-in-a-brightly-lit-stadium-at-image_19326030.webp

Source: pngtree.com

https://media.istockphoto.com/id/2230288887/photo/soccer-player-kicking-ball-on-3d-grass-stadium-field-at-sunset.jpg?
s=612x612&w=0&k=20&c=XBYGMOB0ztZALUy98UB8pfLQFNDh_PbOb6KerLW_xHg=

Source: www.istockphoto.com

https://png.pngtree.com/background/20250110/original/pngtree-evening-cricket-stadium-panorama-with-spectacular-lighting-picture-image_15952382.jpg

Source: pngtree.com

https://png.pngtree.com/thumb_back/fh260/background/20260607/pngtree-golden-world-cup-trophy-on-a-football-field-with-confetti-and-image_21900153.webp

Source: pngtree.com

PREDICTING

EFFICIENCY

By José Pabón PhD

Average vs. Instantaneous Rates of Change in Economics

THE AVERAGE RATE OF CHANGE

OPENSTAX PRECALCULUS 2E — SEC. 1.3

Over an interval $[x_1, x_2]$, the average rate of change measures how much the output changes relative to the input.

$$\text{Average Rate} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Geometric Meaning: This is exactly equivalent to the slope of the *secant line* connecting two points on a curve.

Geometrically, the average rate of change of f from $x = a$ to $x = b$ is the slope of the **secant line** joining the points $P(a, f(a))$ and $Q(b, f(b))$ on the graph of f. See Figure 2.68.

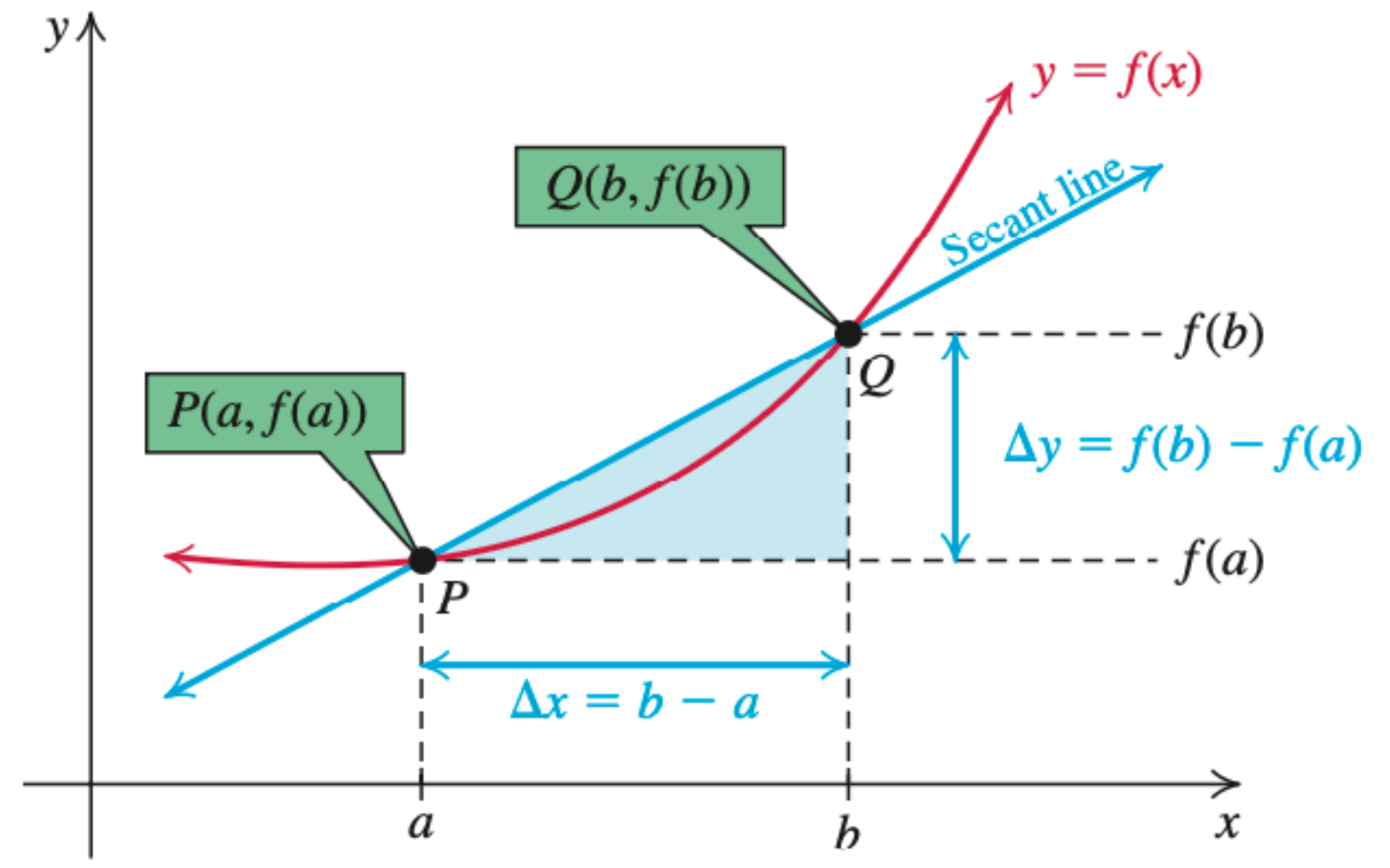


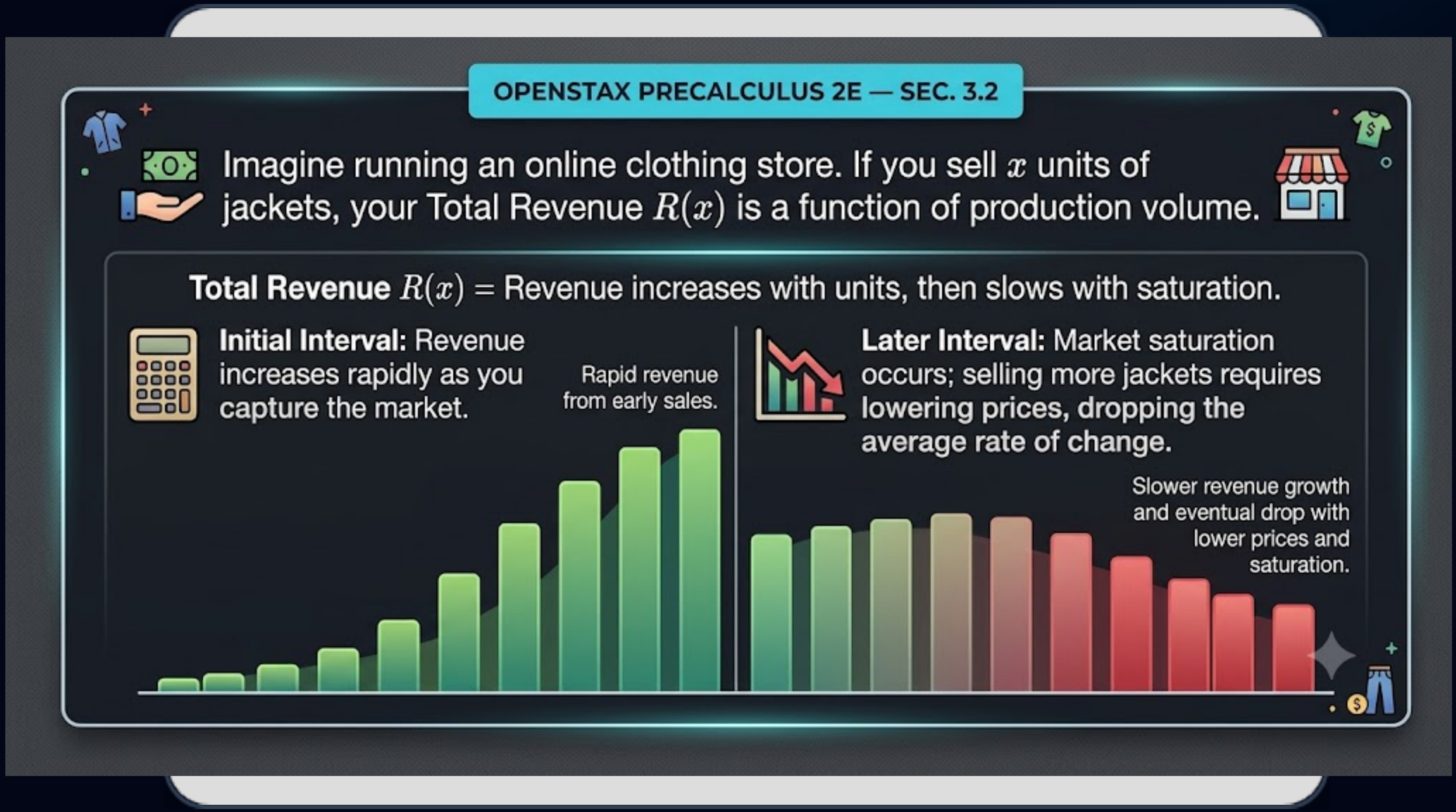
Figure 2.68 ▸ Secant line and average rate of change

APPLICATION: TOTAL REVENUE

OPENSTAX PRECALCULUS 2E — SEC. 3.2

Imagine running an online clothing store. If you sell x units of jackets, your Total Revenue $R(x)$ is a function of production volume.

- **Initial Interval:** Revenue increases rapidly as you capture the market.
- **Later Interval:** Market saturation occurs; selling more jackets requires lowering prices, dropping the average rate of change.



SHRINKING THE INTERVAL: THE DERIVATIVE

OPENSTAX PRECALCULUS 2E — SEC. 12.1 PREVIEW

What if we don't want the average over a week? What if we want to know our efficiency at this **exact instant**?

We shrink the distance between our two points $(h = x_2 - x_1)$ down toward zero:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Difference Quotient

For a function f, the **difference quotient** is

$$\frac{f(x+h) - f(x)}{h}, \quad h \neq 0.$$

Equivalently, the difference quotient can be written in the form $\dfrac{f(x) - f(a)}{x - a}$, $x \neq a$.

This limit gives us the **Instantaneous Rate of Change**, also known as the slope of the *tangent line*.

ECONOMIC EFFICIENCY: MARGINAL ANALYSIS

In economics, instantaneous rates of change are called **marginal quantities**. They tell you the impact of producing just *one more item*.

MARGINAL COST (MC)

The instantaneous rate of change of total cost. How much does item #1001 cost to produce compared to #1000?

MARGINAL REVENUE (MR)

The instantaneous rate of change of total revenue. How much additional cash does selling item #1001 bring in?

FINDING PEAK EFFICIENCY

OPENSTAX PRECALCULUS 2E — SEC. 3.2 MAXIMUMS

To maximize net profit, a company looks at the relationship between Marginal Revenue and Marginal Cost:

- **MR > MC:** Efficiency is high! You make more than it costs. Expand production!
- **MR < MC:** Inefficient. You are losing money on the last few units. Scale back!
- **MR = MC:** Peak profit/efficiency attained. The slope of the profit function is zero.

THE FOUNDATION OF CALCULUS

- **Average Rate (Secant):** Tracks change over an interval.
- **Instantaneous Rate (Tangent):** Tracks change at a single moment.
- **Efficiency:** Optimized when marginal changes balance ($MR = MC$).

READY FOR CALCULUS?

Open up your Precalculus 2e book to explore Section 1.3!