Complex AnalysisMAT656.

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These problems are from the pdf received from Prof. Blackmore. The course textbook is Ablowitz and Fokas [1].

1 August2018 Problem4. - REDACTED

1.1 Proof from Ablowitz and Fokas:

$f(z)$ analytic at z_0 implies the function has a Taylor series expression about $z = z_0$. If it has a zero of order n, we have that:

$$f(z) = (z - z_0)^n g(z).$$

With $g(z)$ analytic and having a Taylor series at $z = z_0$ and $g(z_0) \neq 0$. There must exist a maximum integer n, otherwise $f(z)$ would be identically zero in a neighborhood of z_0 and must vanish everywhere in our domain. This is a result from previous theorems including this theorem (3.2.6 in the text):

Theorem - If $f(z), g(z)$, are analytic in common domain D, if $f(z), g(z)$ coincide in some subportion $D' \subset D$, then $f(z) = g(z) \forall z \in D$.

Given $g(z)$ is analytic, we have that:

$$\exists \;\; \epsilon > 0, \left|g(z) - g(z_0)\right| < \epsilon \text{ whenever z is in a neighborhood of} z_0, 0 < |z - z_0| < \delta.$$

Thus, $g(z)$ can be made arbitrarily as close to $g(z_0)$ as desired, thus:

$$g(z) \neq 0, f(z) \neq 0, \text{ in this neighborhood }.$$

$\therefore$ if f is a nonconstant analytic function in a domain$D \subset C$, then if $f(z_0) = 0 for z_0 \in D$, there exists an$\epsilon > 0$ such that f is not zero for any point of the punctured neighborhood✓.

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2 Conclusion

Thank you to Prof. Blackmore for his instruction, lectures and office hours effort. I look forward to any feedback and learning more of the material in this course and beyond.

References

[1] Mark J Ablowitz, Athanassios S Fokas, and AS Fokas. *Complex variables: introduction and applications*. Cambridge University Press, 2003.